PROJECT

WAVES & SIMPLE HARMONIC MOTION

“EVERY WAVE, REGARDLESS OF HOW HIGH AND FORCEFUL IT CRESTS, MUST EVENTUALLY COLLAPSE WITHIN ITSELF.”

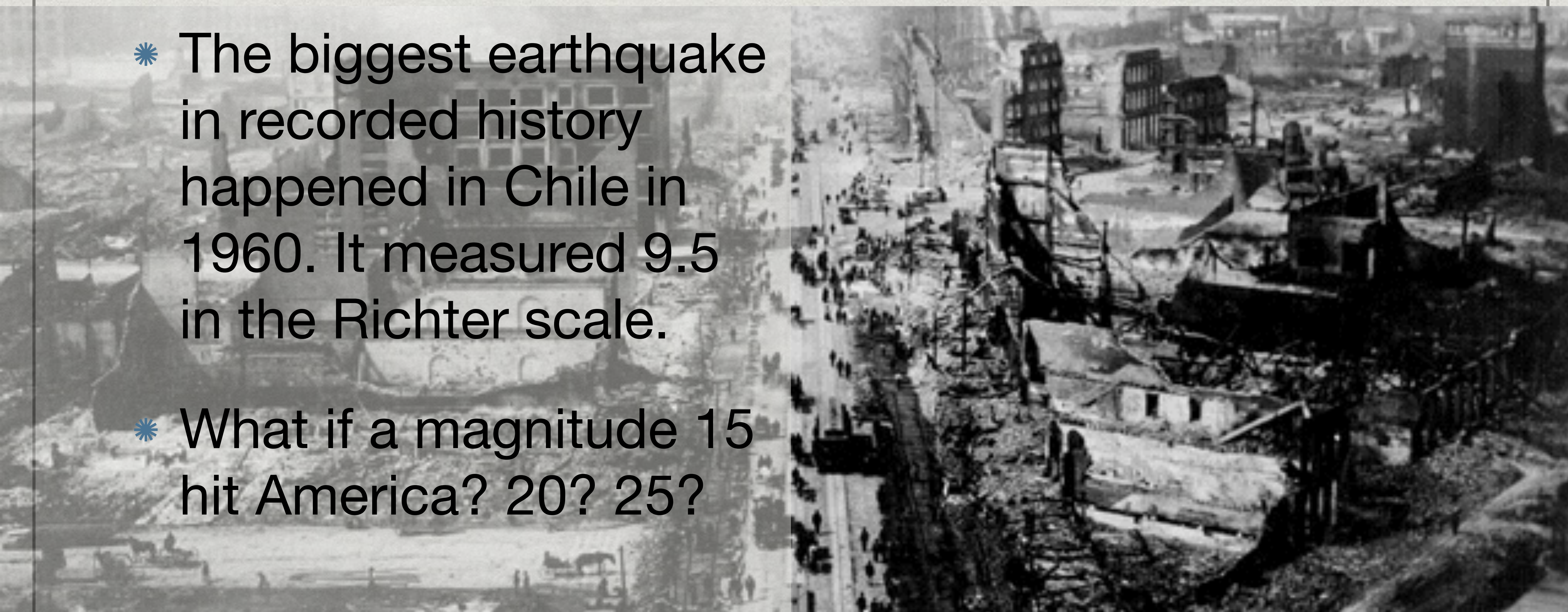
- STEFAN ZWEIG

What's a Wave?

- A wave is a wiggle in time and space
- The source of a wave is almost always a vibration
  - A vibration is a wiggle in time
- So a wave is basically a traveling vibration
  - BUT carries *energy* from the vibrating source to the receiver; it does NOT transfer matter


SF 14
JAX
2ND & GOAL
FOX
FANTASY TE 4. S. McGrath 2.20 PTS (REC, 22 YDS)
RedZone

Richter 15



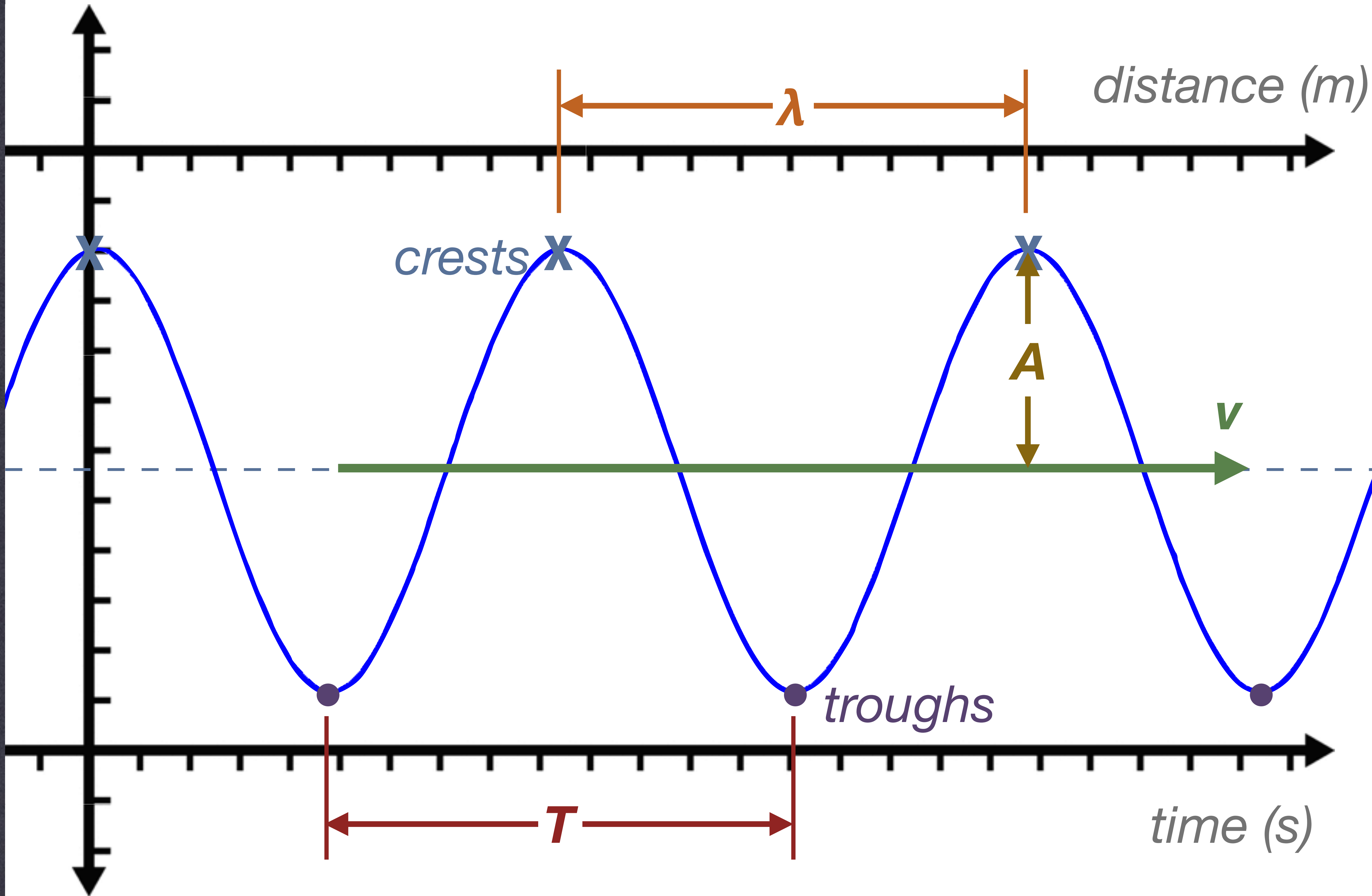
- The biggest earthquake in recorded history happened in Chile in 1960. It measured 9.5 in the Richter scale.
- What if a magnitude 15 hit America? 20? 25?

Qualities of a Wave

- Period (T)
  - Time it takes for one back-and-forth cycle
  - In seconds (s)
- Wavelength (λ)
  - Distance between successive identical parts of the wave
  - In meters (m)
- Frequency (f)
  - Number of vibrations in a given time
  - In Hertz (Hz)
  - $f = 1/T$

Qualities of a Wave

- Velocity
  - Speed and direction of the wave
  - In m/s
  - $v = \lambda f$
- Crests
  - Peaks or high point of the wave
- Troughs
  - Valleys or low points of the wave
- Amplitude (*A*)
  - Distance from midpoint to crest (or trough)
  - Maximum displacement from equilibrium



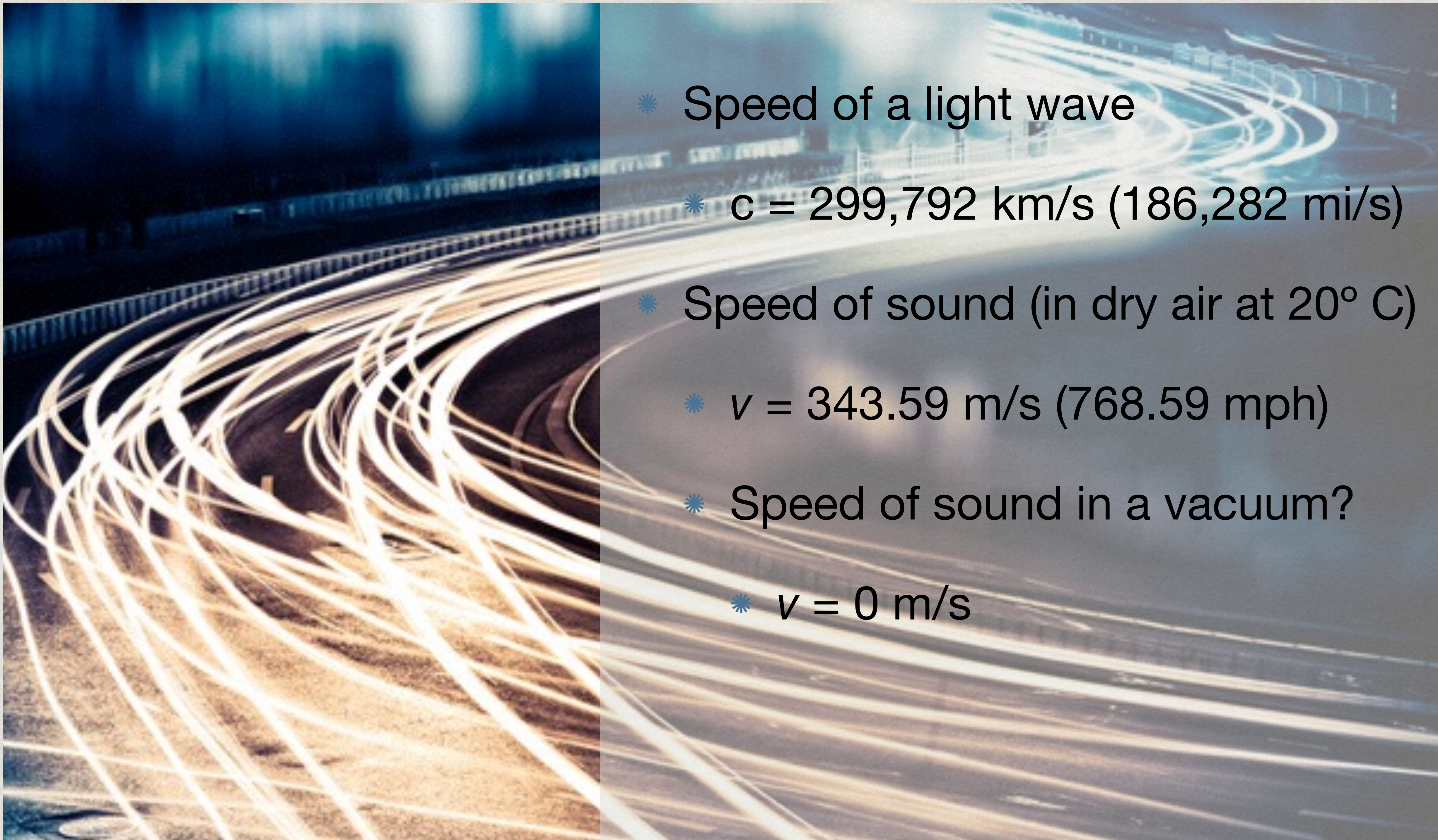
QUALITIES OF A WAVE

WAVE MOTION

Wave Speed

- In a freight train, each car is 10 m long. If two cars roll by you every second, how fast is the train moving?
  - v = d/t = 2x(10 m)/(1 s) = 20 m/s
- A wave has a wavelength of 10 m. If the frequency is 2 Hz, how fast is the wave traveling?
  - $v = \lambda f$ = (10 m)(2 Hz) = 20 m/s

Wave Speed



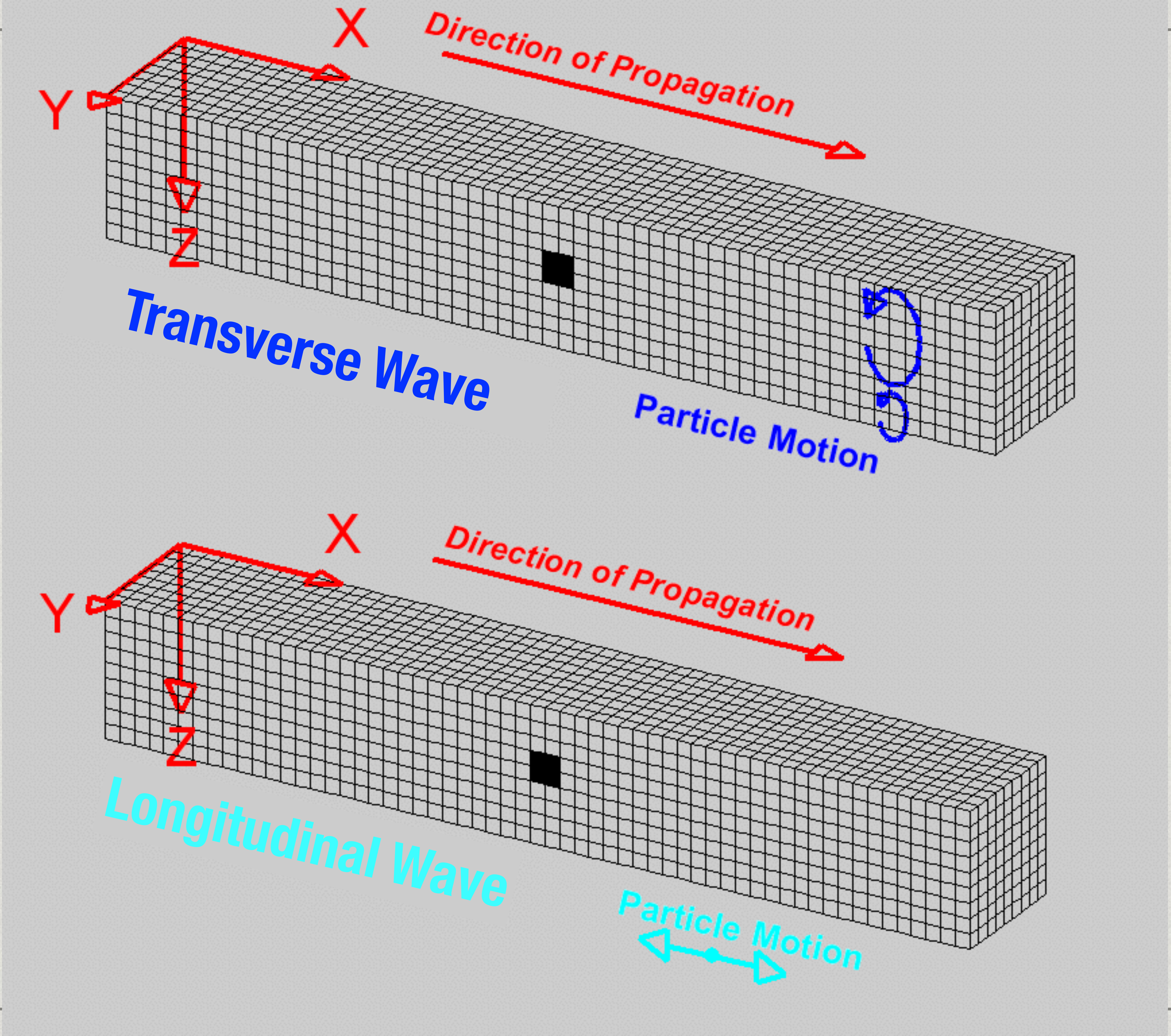
- Speed of a light wave
  - c = 299,792 km/s (186,282 mi/s)
- Speed of sound (in dry air at 20° C)
  - *v* = 343.59 m/s (768.59 mph)
  - Speed of sound in a vacuum?
    - *v* = 0 m/s

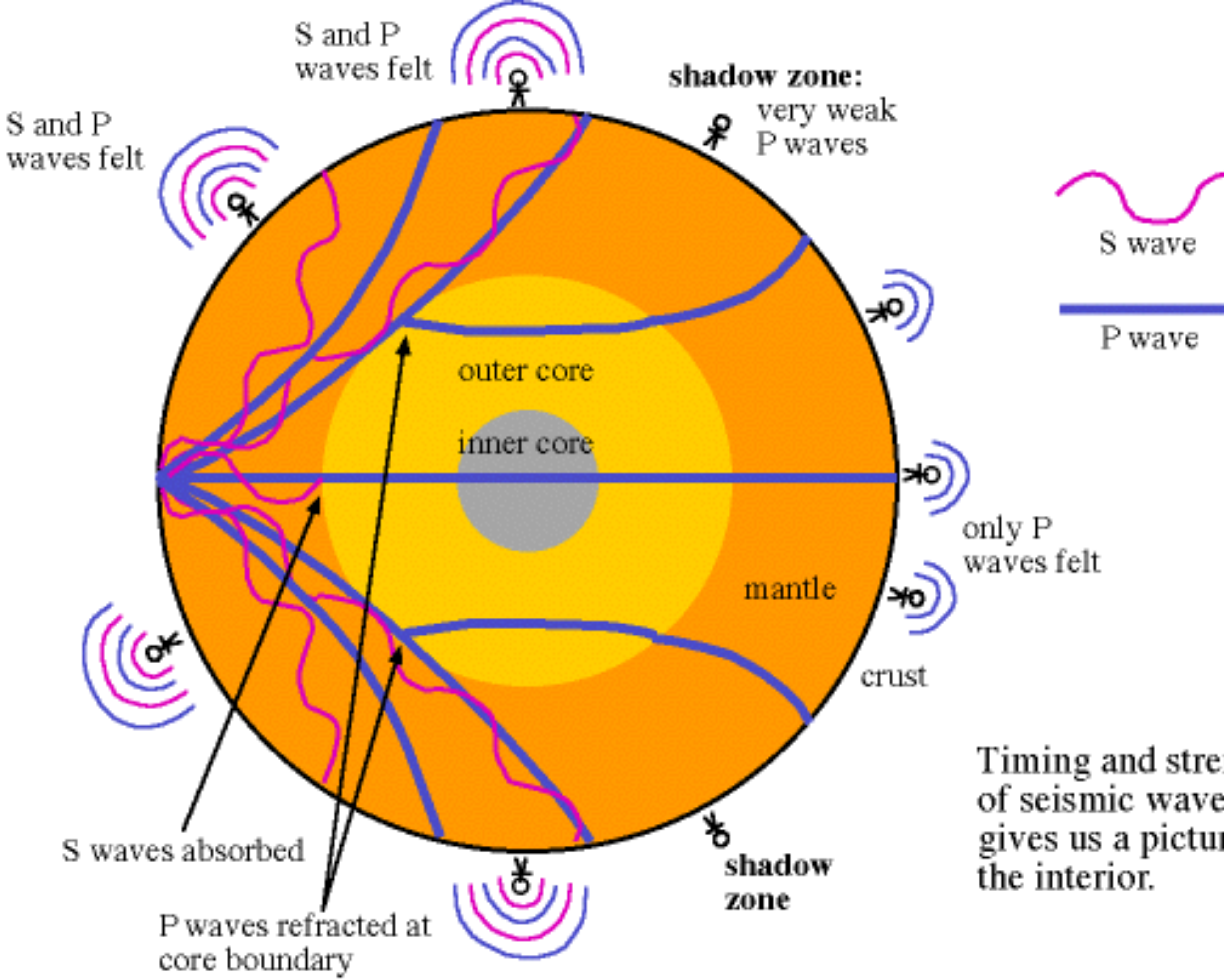
Types of Waves

- Transverse Waves
  - Motion of the medium is perpendicular to the direction in which the wave travels
  - Examples:
    - Ripples in the water
    - A whip
    - Light
    - Earthquake secondary waves

Types of Waves

- Longitudinal Waves
  - Motion of the medium is in the same direction as in which the wave travels
  - Also called *compression waves*
  - Examples:
    - Earthquake primary waves
    - Sound


X
Y
Z
Direction of Propagation
Transverse Wave
Particle Motion
X
Y
Z
Direction of Propagation
Longitudinal Wave
Particle Motion



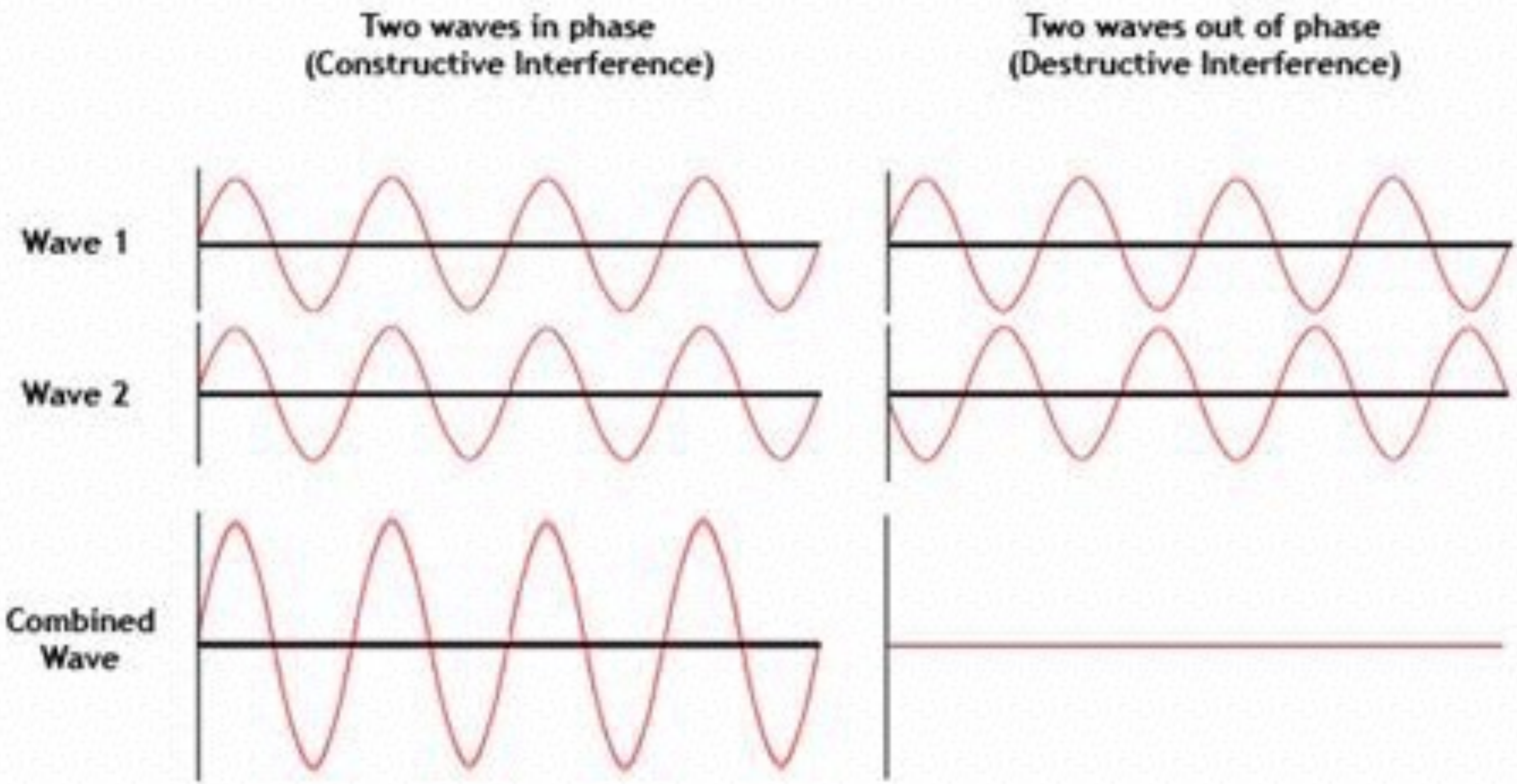
Timing and strength of seismic waves gives us a picture of the interior.

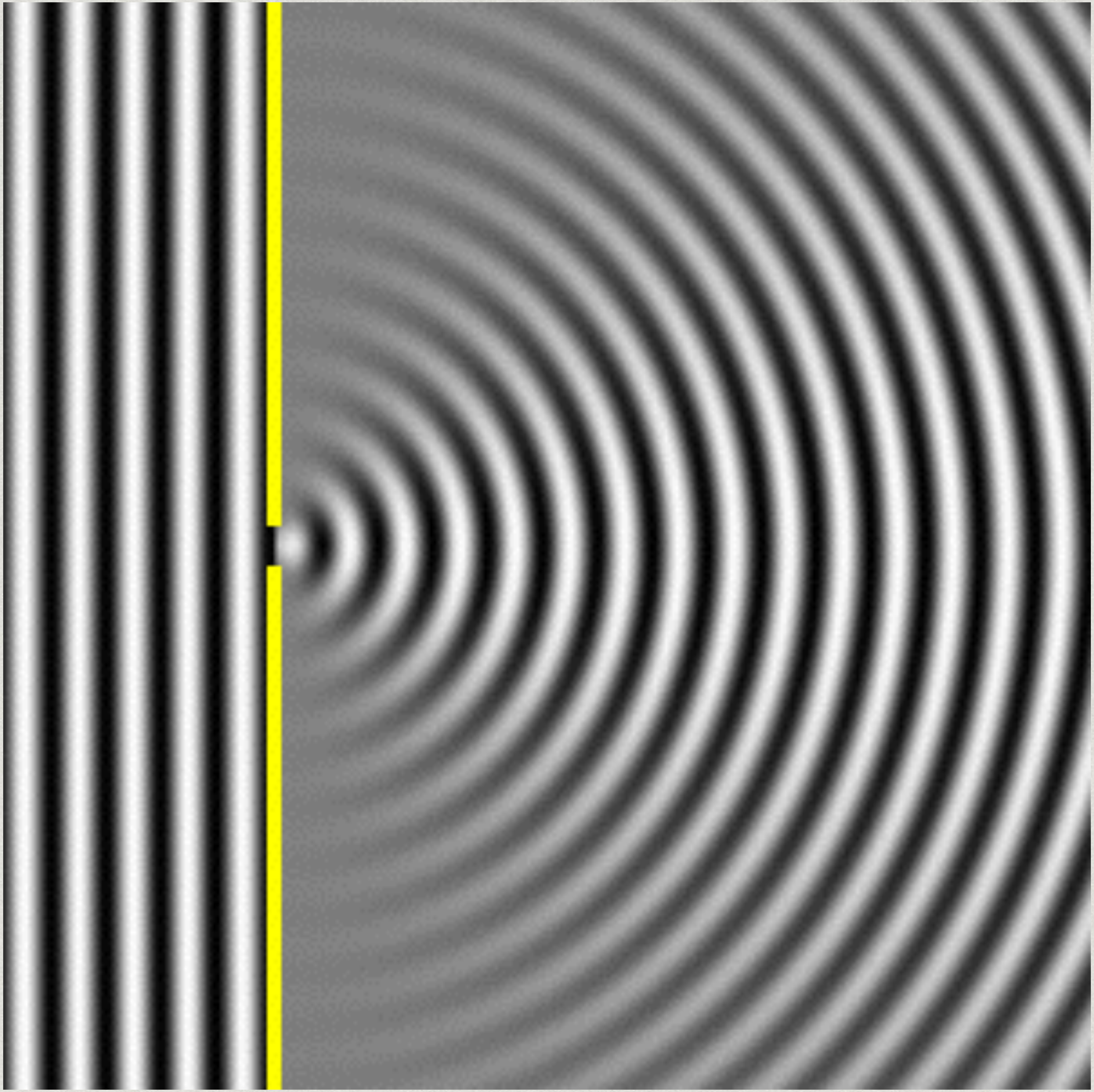
Interference

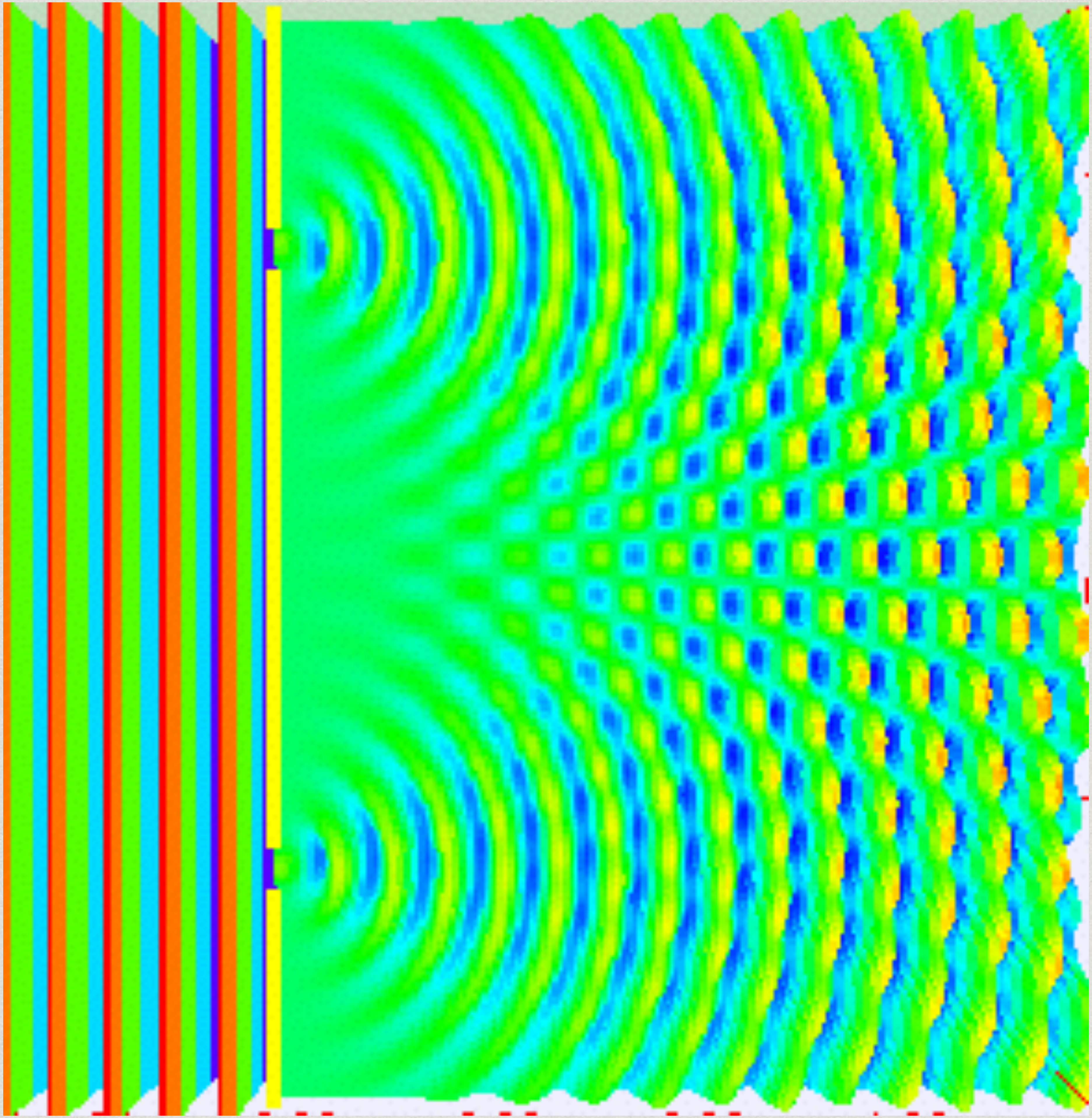
- Occurs when two of more waves meet
- Parts of the waves may overlap and form an *interference pattern*
  - Wave effects may be increased, decreased, or neutralized

Interference

- When the crest of one wave overlaps with the crest of another, their individual effects add up
  - Called *constructive interference*
- When the crest of one wave meets the trough of another, their individual effects decrease
  - Called *destructive interference*
- Characteristic of *all* wave motion, whether water waves, sound waves, or light waves

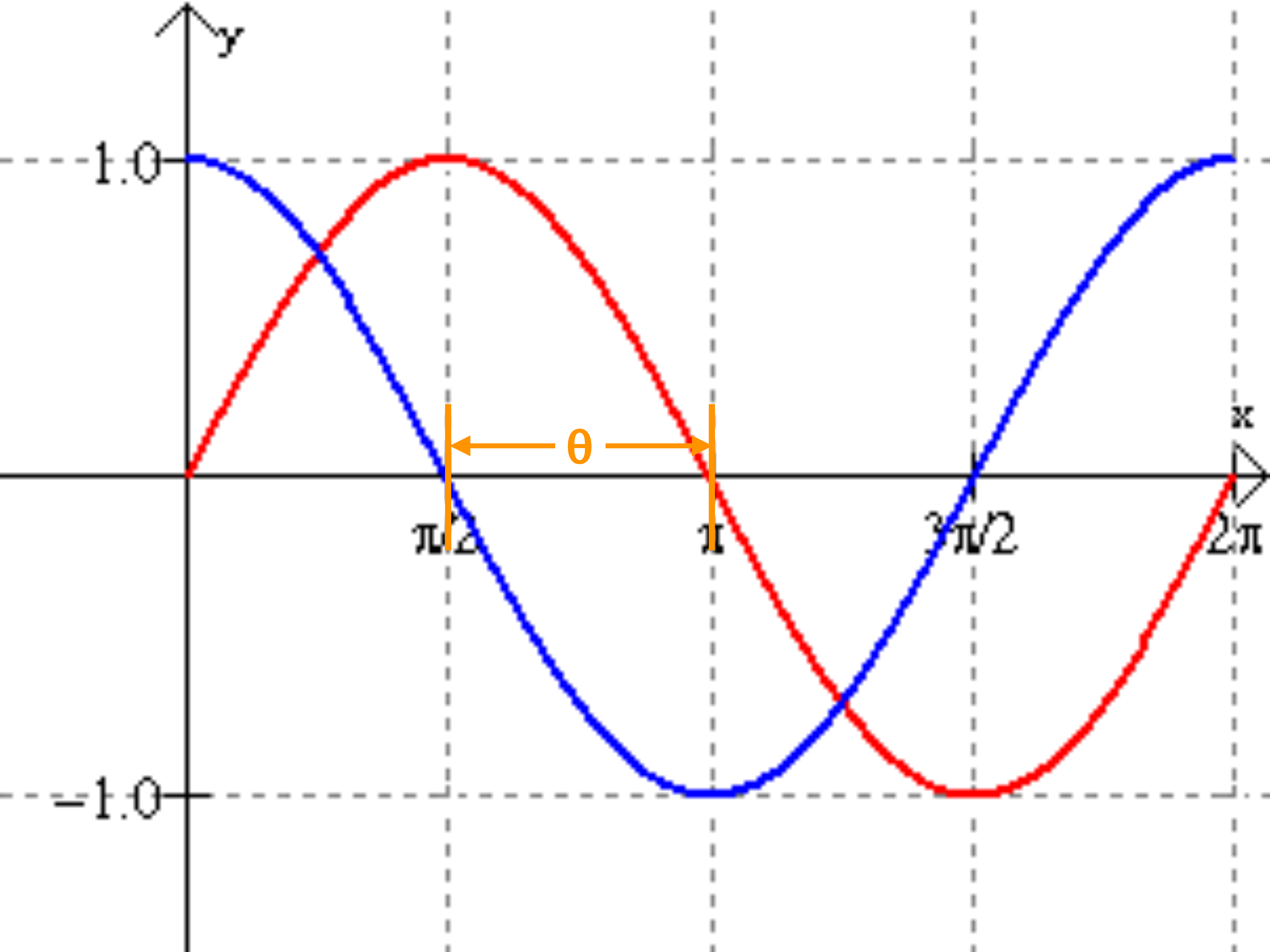

Two waves in phase
(Constructive Interference)
Two waves out of phase
(Destructive Interference)
Wave 1
Wave 2
Combined
Wave





Phase

* Phase is the relationship between the period of a wave and an external reference point
* Two waves which are *in phase* are in synch
* Two waves which are *out of phase* are out of synch

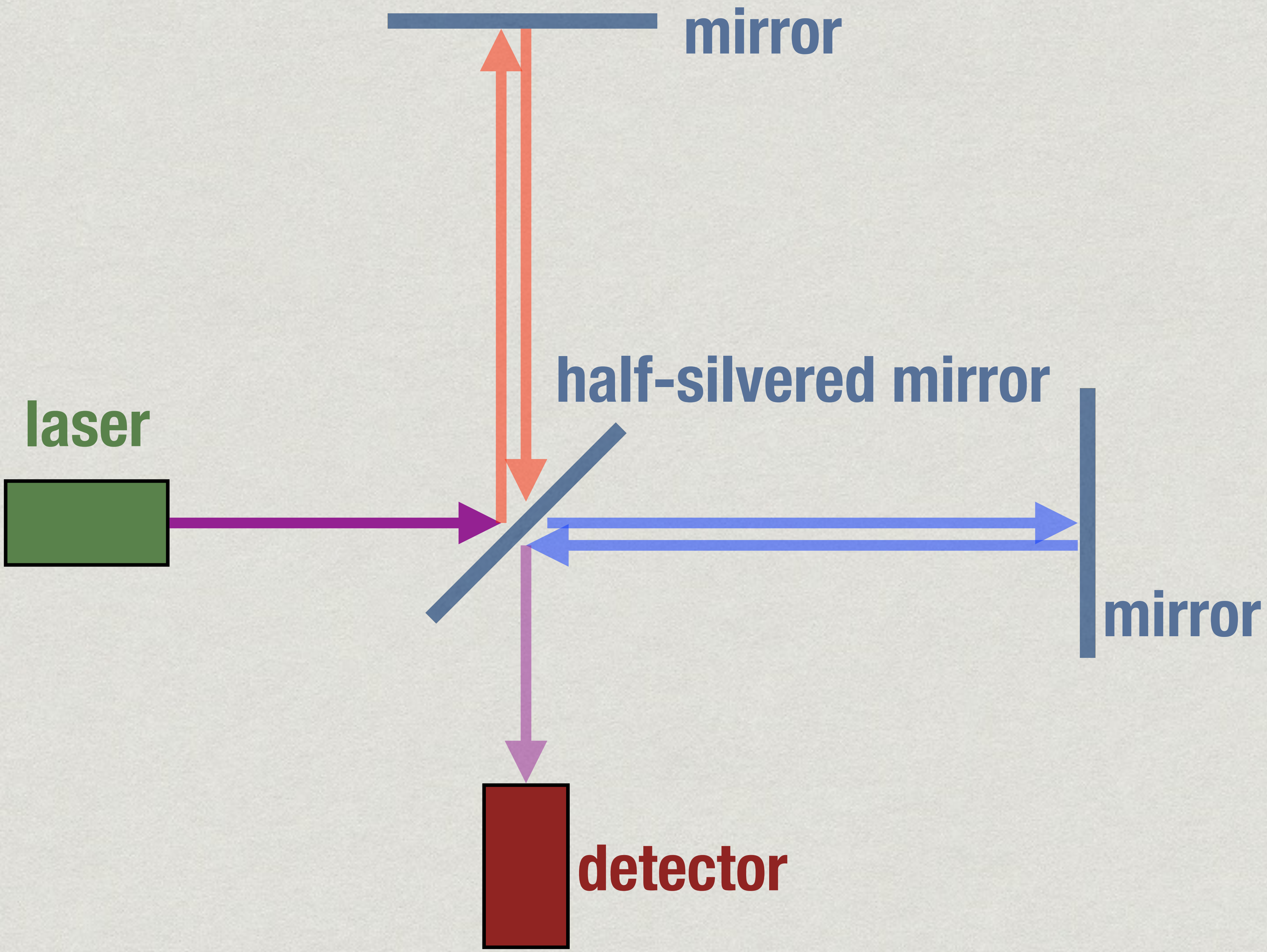


y
1.0
−1.0
x
π/2
π
3π/2
2π
θ

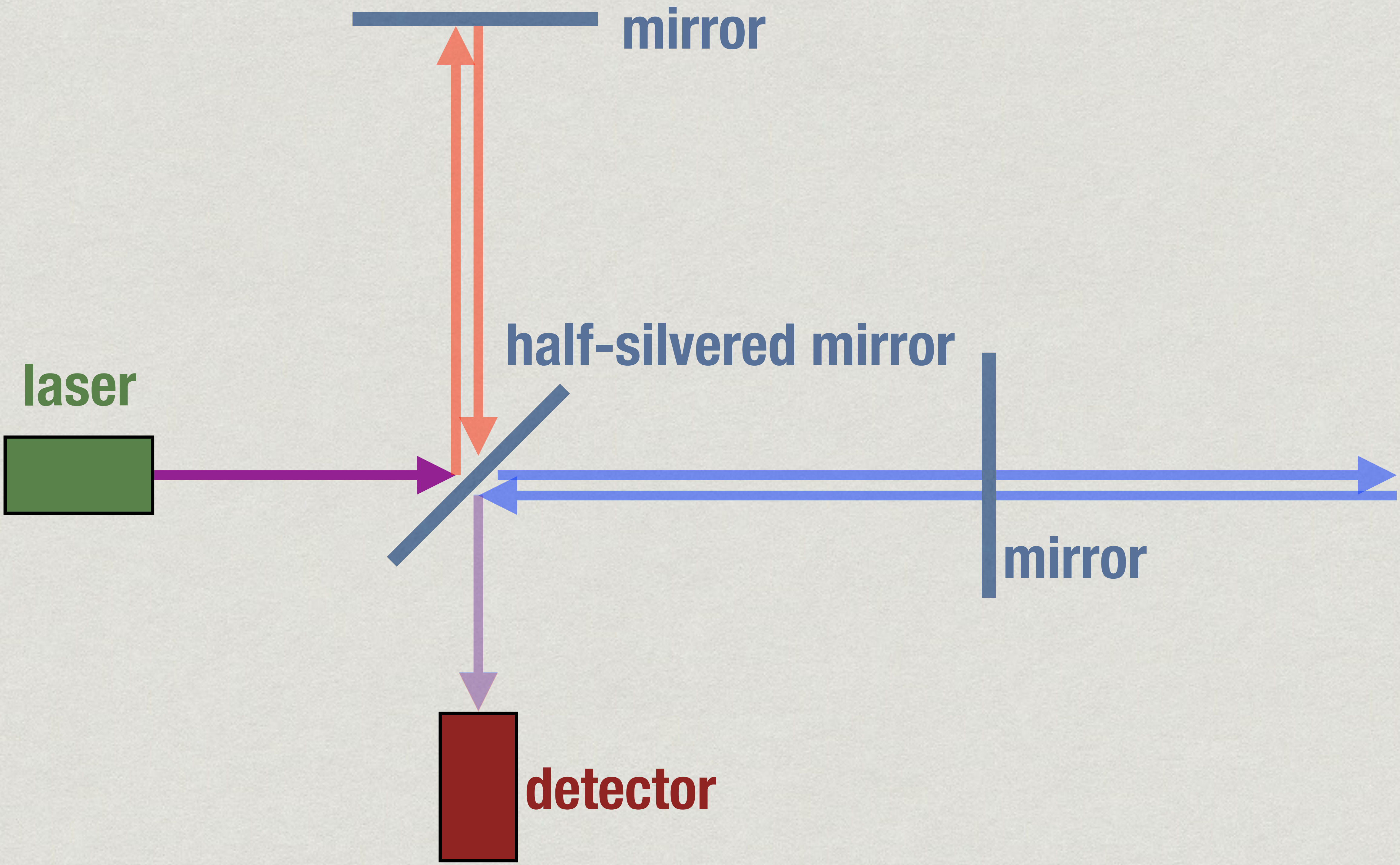
Interferometry

- A family of techniques in which you use wave interference patterns to extract information about the wave is called interferometry
  - Usually measures interference between light waves (especially from lasers)

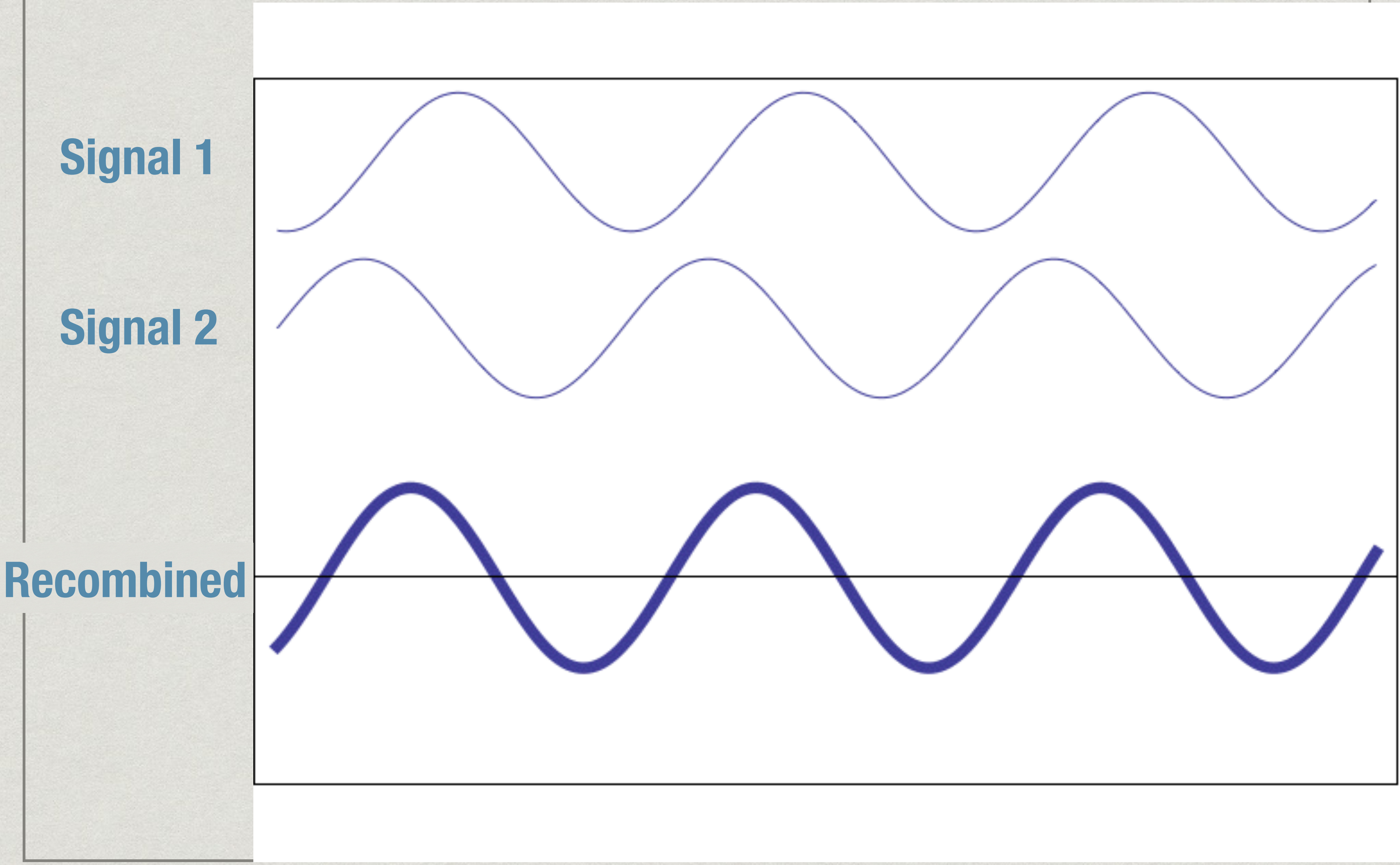
Interferometry

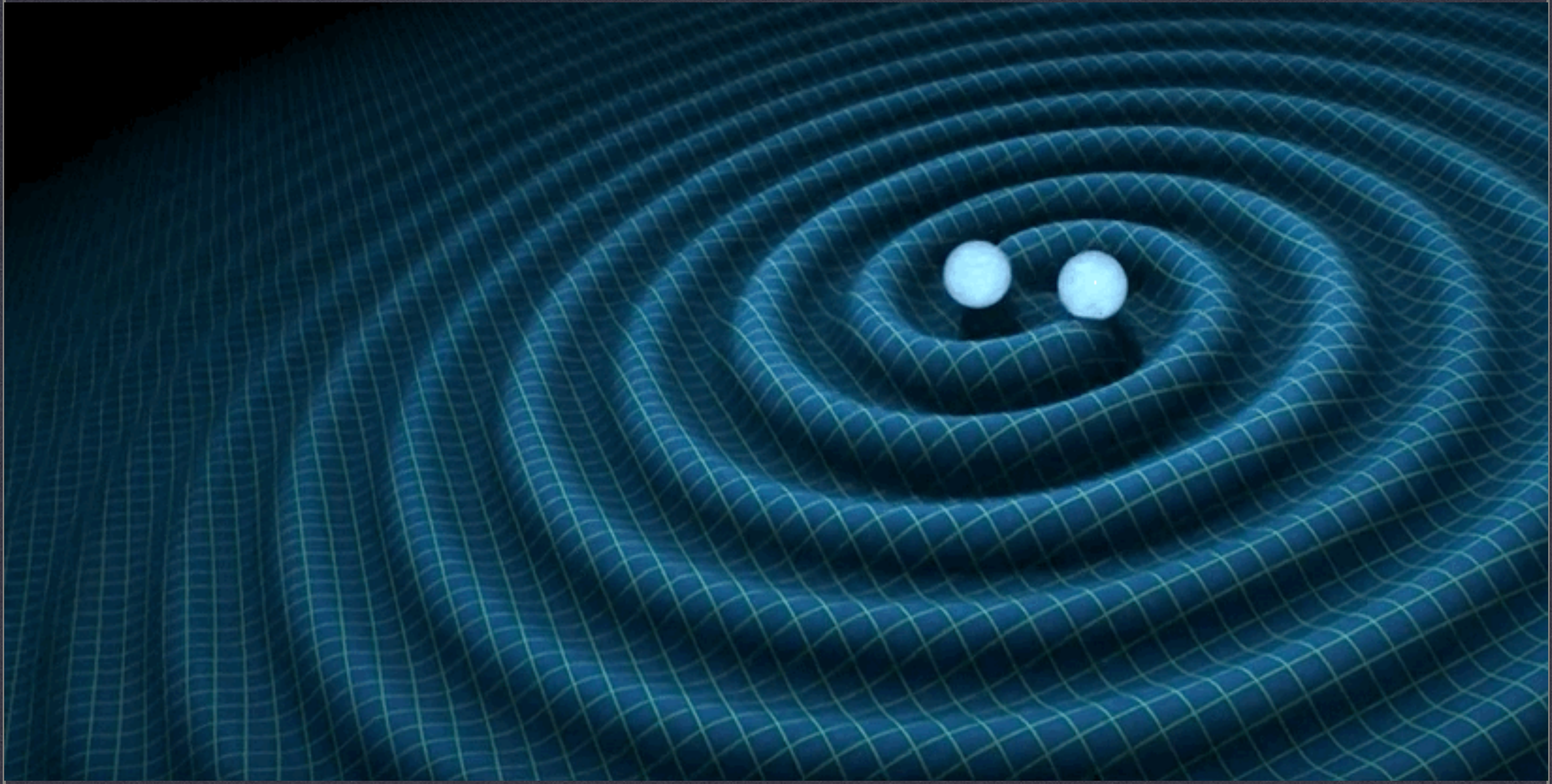


Interferometry



Interference





GRAVITATIONAL WAVES

FROM TWO ORBITING BLACK HOLES

GRAVITATIONAL WAVES & INTERFEROMETRY

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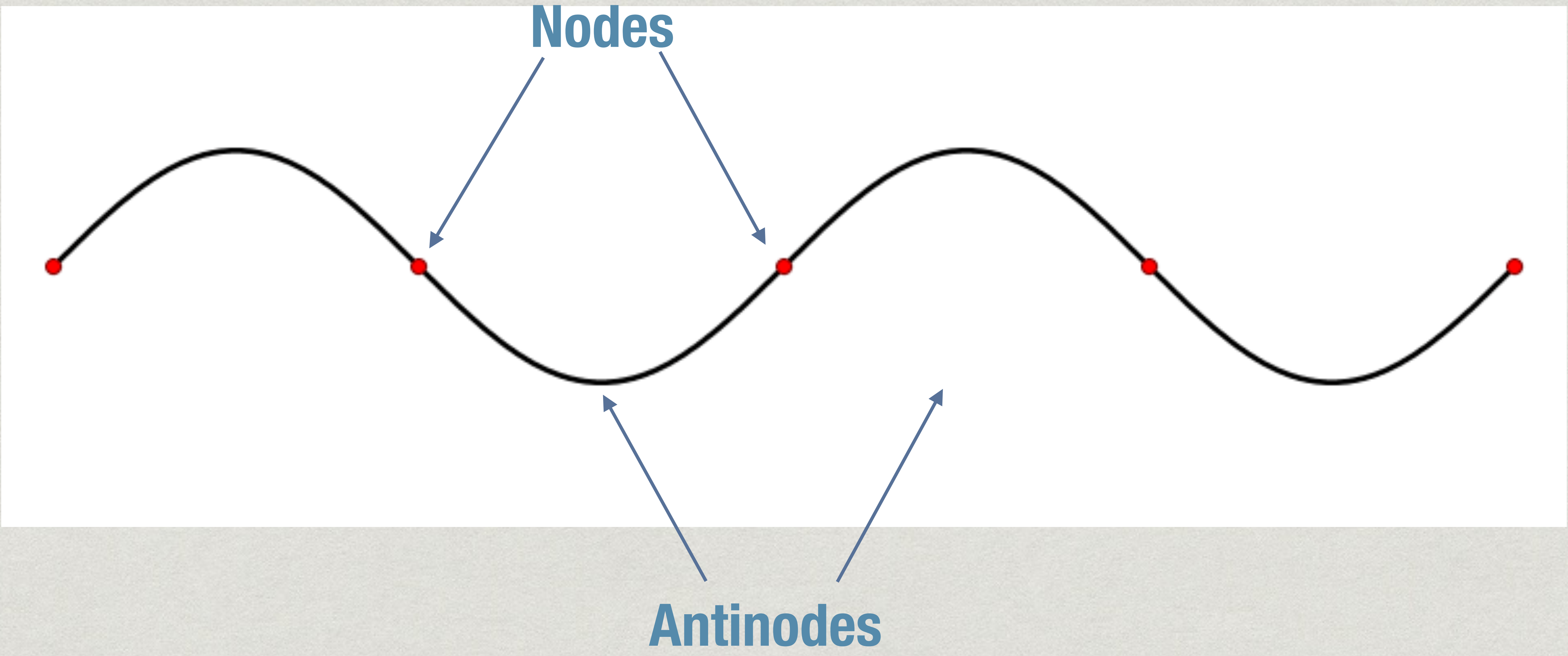
Standing Waves

- Also known as a *stationary wave*
- A standing wave is one where particular points on the wave are “fixed,” or stationary
  - Fixed points on a standing wave are called *nodes*
  - Positions on a standing wave with the largest amplitudes are called *antinodes*
    - Antinodes occur halfway between nodes

Standing Waves



Standing Waves

- Standing waves are the result of interference
  - Two waves of equal amplitude and wavelength pass through each other in opposite directions
  - Waves are always out of phase at the nodes
  - Nodes are are stable regions of destructive interference

ACOUSTIC LEVITATION

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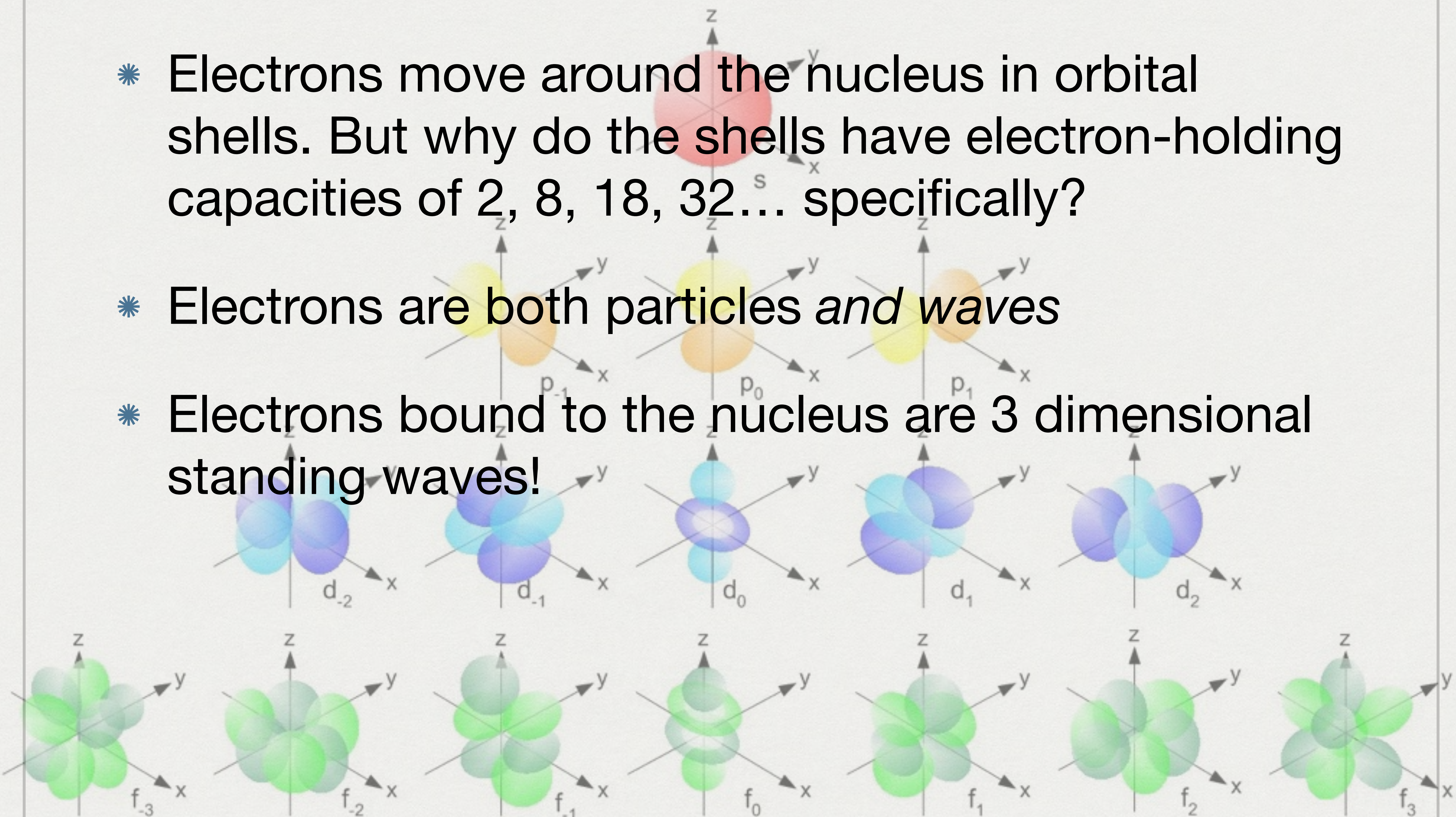
2D STANDING WAVES

HTTPS://WWW.YOUTUBE.COM/WATCH?V=WVJAGRUBF4W

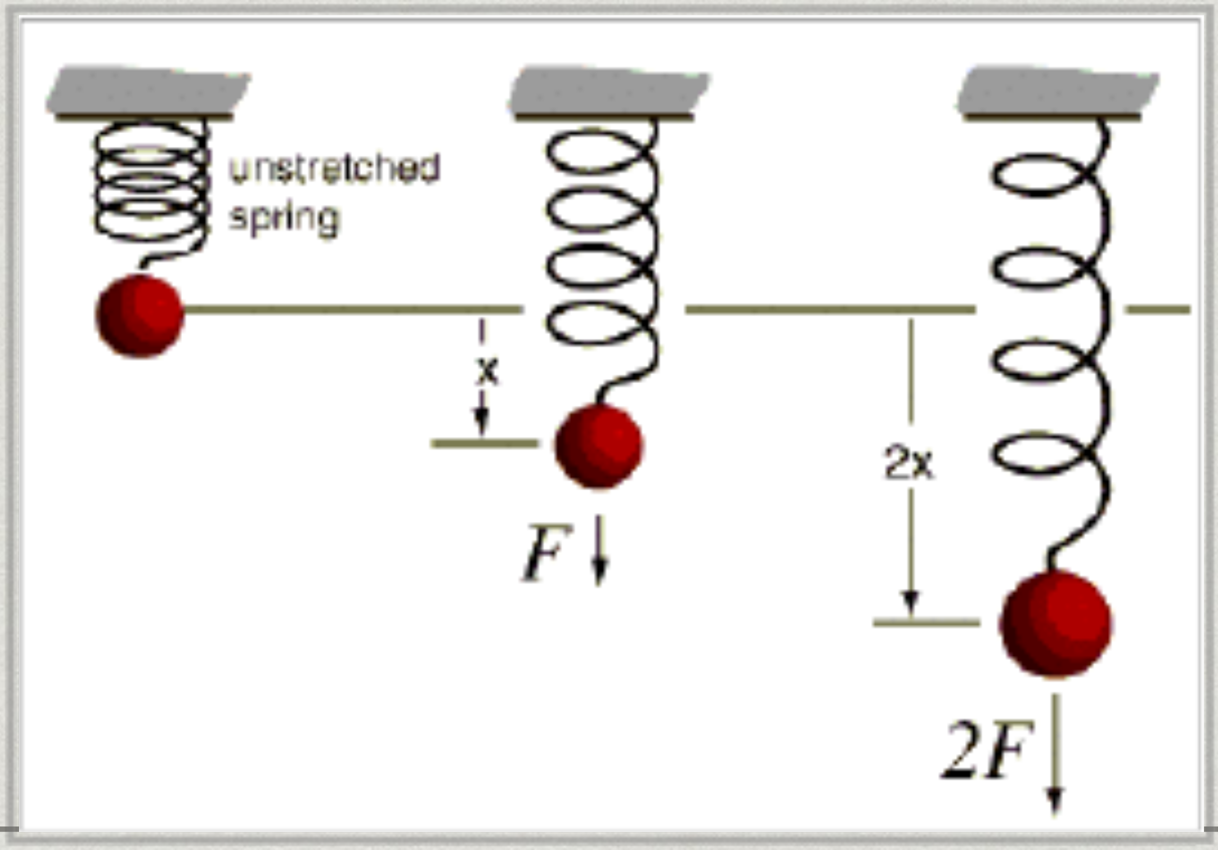
Electron Orbitals

- Electrons move around the nucleus in orbital shells. But why do the shells have electron-holding capacities of 2, 8, 18, 32… specifically?
- Electrons are both particles *and waves*
- Electrons bound to the nucleus are 3 dimensional standing waves!



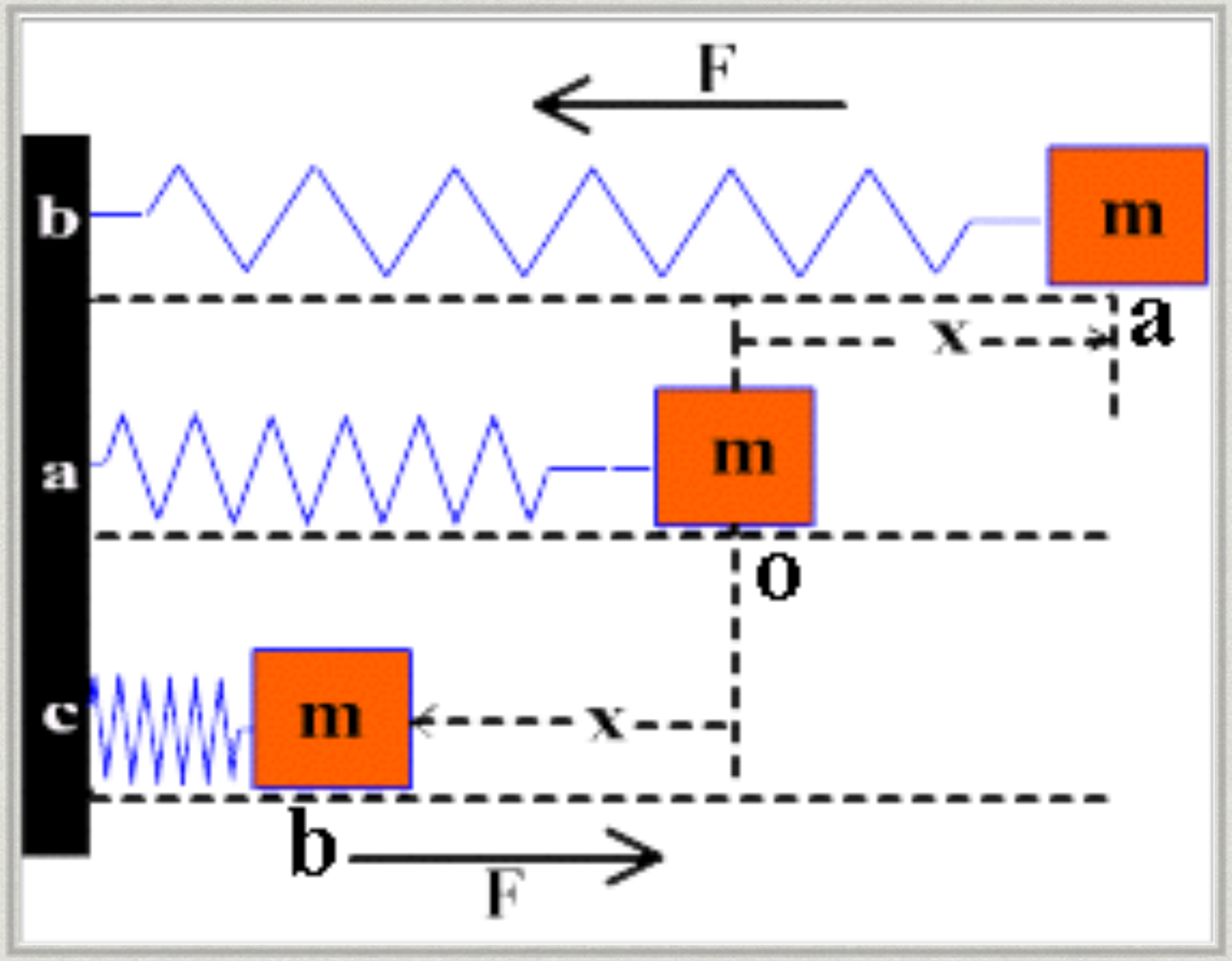
Simple Harmonic Motion

- Oscillatory motion under a restoring force proportional to the amount of displacement from equilibrium
- A restoring force is a force that tries to move the system back to equilibrium



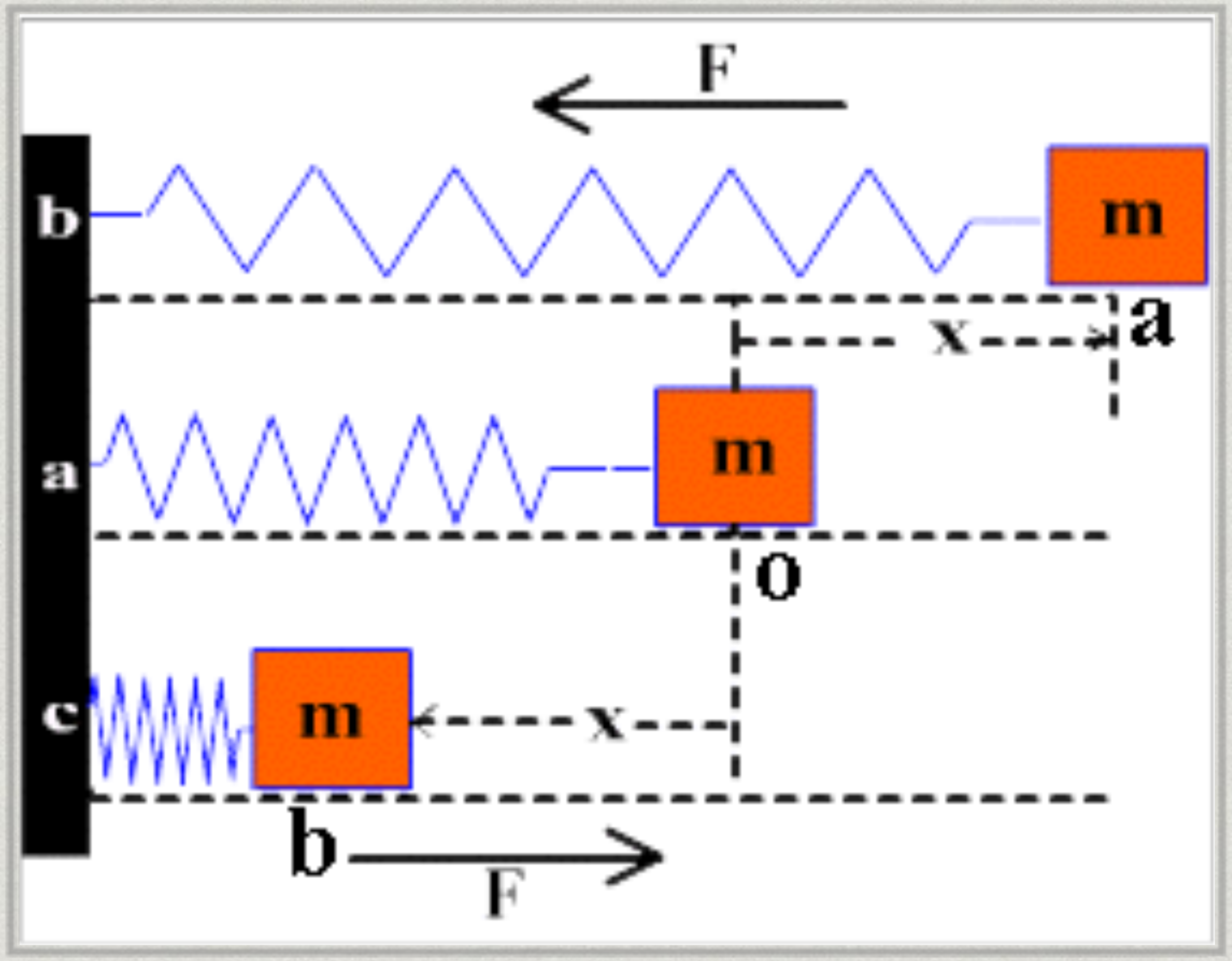
Describing Oscillation

- Begin with the origin at equilibrium
- x is then the displacement from equilibrium, i.e. the change in the length of the spring
- When displaced, the spring force tends to restore the mass to equilibrium — *restoring force*
- Oscillation can only occur when there is a restoring force



Describing Oscillation

- Displace the body to the right to $x = A$ and let go
- Net force and acceleration are to the __**left**__
- Reaches maximum velocity at __*O*__
- Net force at *O* is __**zero**__
- Overshoots and compresses spring
- Net force and acceleration to __**right**__
- Compresses until $x =$ __***-A***__



Terms for Periodic Motion

- Amplitude (A) — magnitude of displacement from equilibrium
  - Total range of motion is $2A$
- Period (T) — seconds per cycle; $T = 1/f$
- Frequency (f) — cycles per second; $f = 1/T$
- Angular frequency (ω) — $\omega = 2\pi f = 2\pi/T$
  - essentially average angular speed

Simple Harmonic Motion

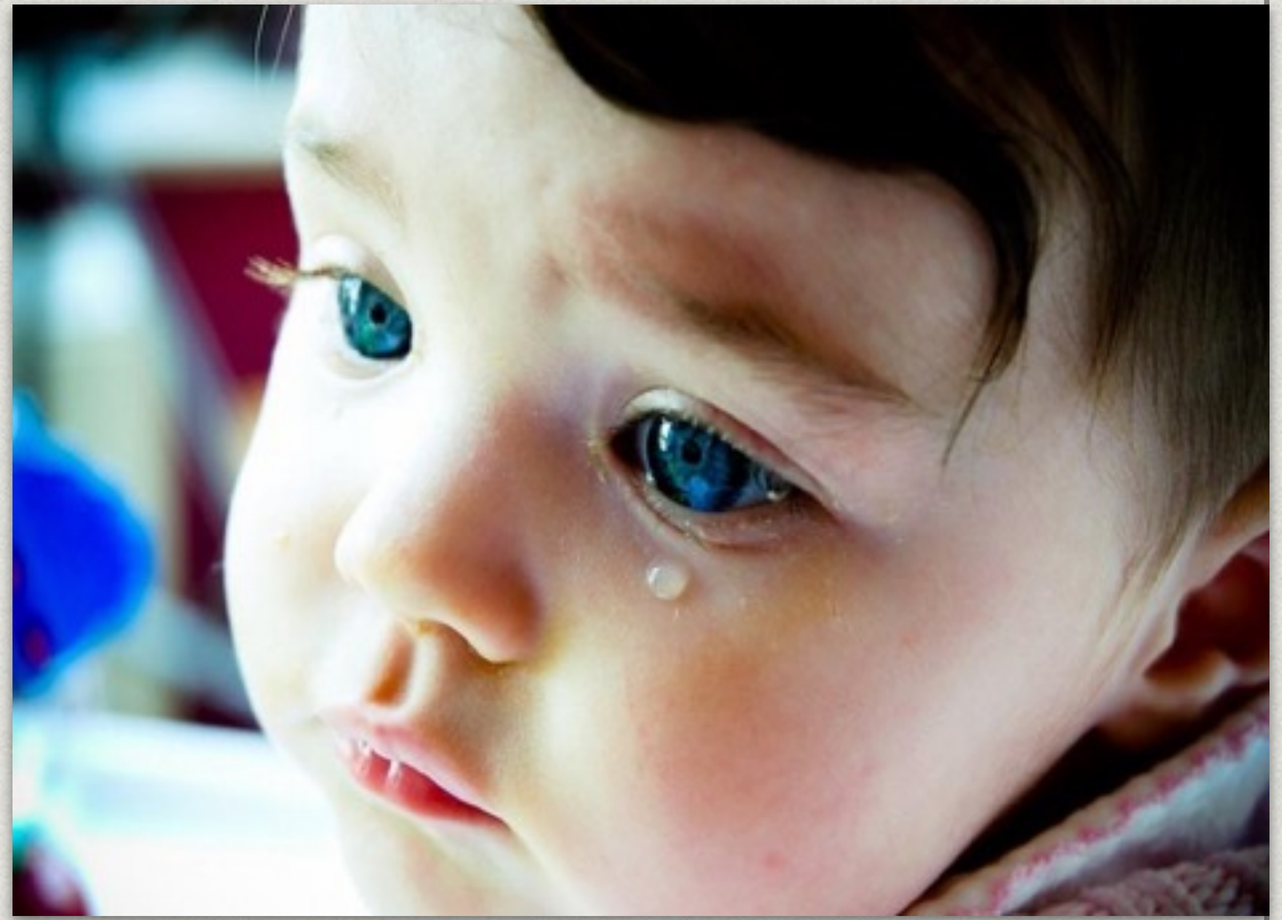
- Periodic motion where the restoring force is directly proportional to the displacement from equilibrium
- Typified by motion obeying Hooke's Law
- Motion is sinusoidal

Hooke's Law

- $F_s = -kx$
  - Restoring force exerted by an ideal spring
- IMPORTANT ASSUMPTION: no friction and massless spring
- $a = F_s/m = -kx/m$

Things to Note

- $a = -kx/m$
- Acceleration is NOT constant
- Cannot use kinematic equations



Harmonic Oscillators

- Why are they important?
- SHM is a powerful model for many different periodic motions
  - E.g. Electric current in an AC circuit, vibrations on a guitar string, vibrations of atoms in molecules

Circle of Reference

- Imagine watching the shadow of a ball being swung in a vertical circle
- If the amplitude of the body's oscillation is equal to the radius of the sting attached to the ball
- And if the angular frequency $2\pi f$ of the oscillating body is equal to the angular speed ω of the revolving ball
- The motion of the shadow is *identical* to the motion of the ideal spring
- http://www.animations.physics.unsw.edu.au/jw/SHM.htm

SHM and Circular Motion

- acceleration of ball on string: $a = \omega^2 r$
- acceleration of S-M system: $a = (k/m)x$
    - $r = x$
- $\omega^2 = k/m$ or $\omega = \sqrt{(k/m)}$

exactly equal

SHM and Circular Motion

- $f = \omega/2\pi = (1/2\pi)\sqrt{(k/m)}$
- $T = 2\pi/\omega = 2\pi\sqrt{(m/k)}$
  - The larger the mass, the greater the moment of inertia, the longer it takes to complete a cycle
  - The stiffer the spring (the larger *k*), the shorter the time *T* per cycle

Things to Note

- Don't confuse frequency f with angular frequency

  $\omega = 2\pi f$

- The period and frequency do NOT depend on the amplitude A

  - Bigger A → larger restoring force → higher average velocity

Example

- A 6.0 N weight is hung from a spring. The weight stretches the spring 0.030 m.
  - Calculate the spring constant *k*
- $F_s = F_g$
- $-kx = -W$
- $k = W/x$
- k = 200 N/m

Example

- The same spring is placed horizontally with one attached to the wall and the other attached to a 0.50 kg mass. The mass is pulled a distance of 0.020 m, released, and allowed to oscillate
    - Find the angular frequency, frequency, and period of oscillation
- $\omega = \sqrt{(k/m)} = \sqrt{(200/0.50)} = 20$ rad/s
- $f = \omega/(2\pi) = 3.18$ Hz
- $T = 1/f = 1/3.18 = 0.314$ s

SHM of a Simple Pendulum

- Almost exactly the same behavior as the mass-spring system
- Except:
  - The restoring force is the force of gravity
  - $T = 2\pi\sqrt{(L/g)}$
    - Independent of *A* (same reason as M-S system)
    - Independent of *m* (guesses why?)

Displacement in SHM

- $x(t) = A\cos(\omega t)$
  - Could have written in terms of sine since $\cos\theta = \sin(\theta + \pi/2)$
  - In SHM the position is a periodic, sinusoidal function of time

Velocity and Acceleration in SHM

- $v(t) = x' = -\omega A \sin(\omega t)$
- $v(x) = -\sqrt{\omega(A^2 - x^2)}$
- $a(t) = x'' = -\omega^2 A \cos(\omega t)$
- $v_{max} = A\omega$

Energy in SHM

- $PE_s = ½kx^2$
- $PE_g = mgh$
- $KE = ½mv^2$

Question

- A 17.0 g mass on a 35 N/m spring is pulled 20 cm from equilibrium and released. What is the position of the mass at time t = 1.2s?

Answer

- $x = A\cos(\omega t)$
  - $\omega = \sqrt{k/m}$
  - $\omega = \sqrt{(35\ \text{N/m})/(0.017\ \text{kg})}$
  - $\omega = 45.4$ rad/s
- $x = (.2\ \text{m})\cos[(45.4\ \text{rad/s})(1.2\ \text{s})]$
- $x = -0.101$ m or -10.1 cm

Question

- You find yourself on a strange planet armed only with a simple pendulum. The bob of the pendulum hangs on a 0.45 m long string and will swing through a full oscillation in 1.7 s once set in motion. Use this information to find the acceleration due to gravity on this foreign planet.