Simple Harmonic Motion

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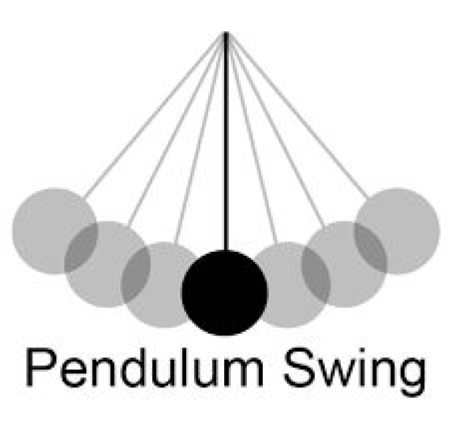
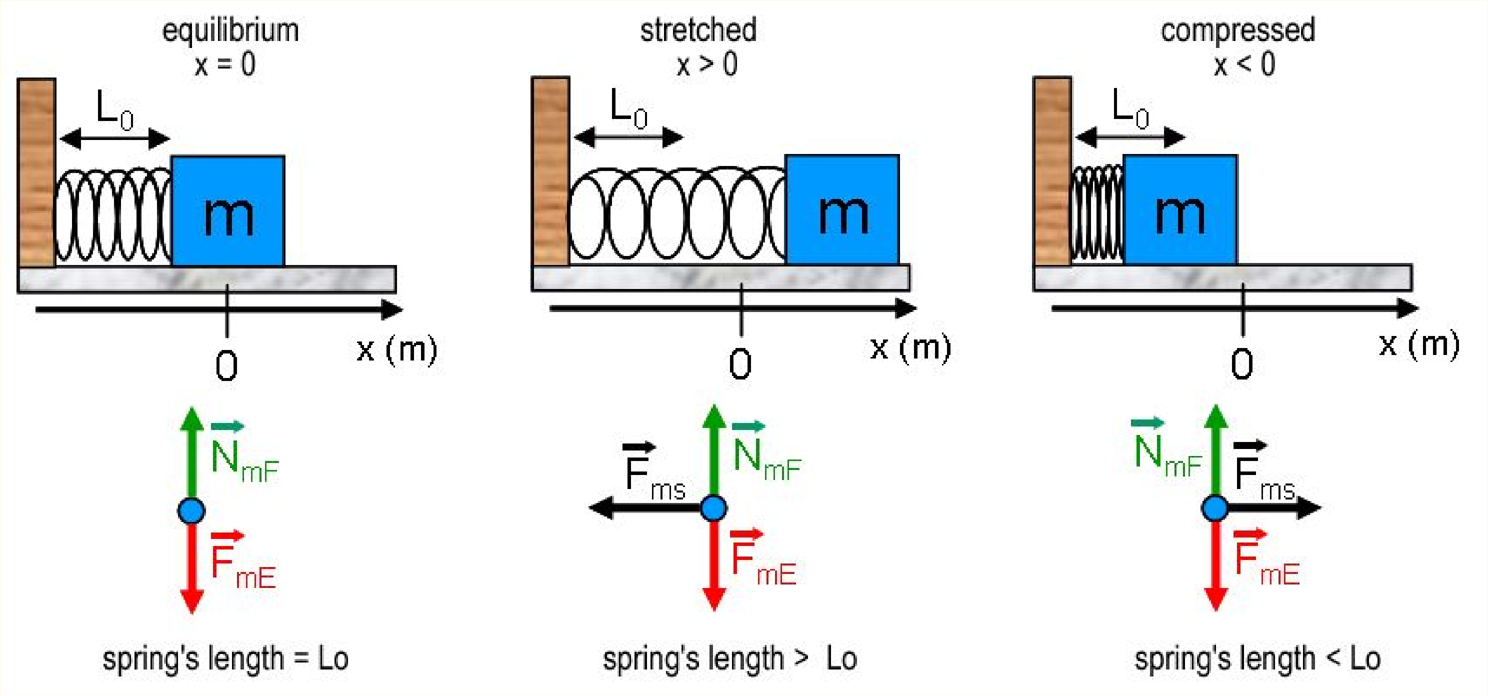
What is Simple Harmonic Motion?

- Simple Harmonic Motion is motion in an oscillatory pattern in which there is a restoring force. It is proportional to the amount of displacement from equilibrium.
- Restoring Force: The force which *restores* the system back to equilibrium.
- There are two particular systems
  - Mass- spring system
  - Simple pendulum



Oscillation



Definitions

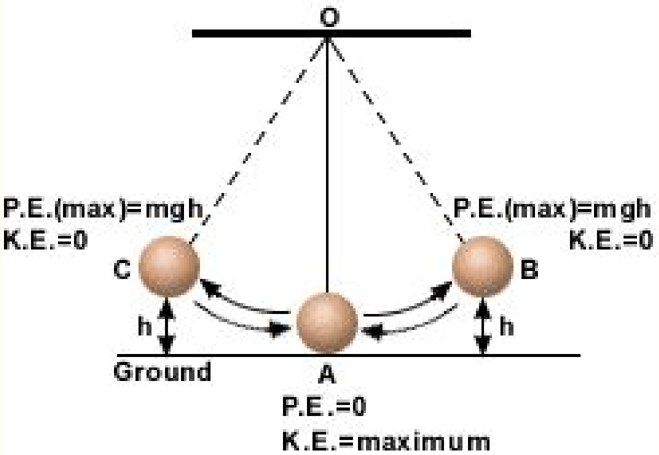
- x= displacement- the distance a system is from equilibrium (measured in m)
- f= frequency- the number of rotations in a given time (measured in Hz)=1/T
- T= Period- the time it takes to complete one rotation (measured in s)=1/f
- ω= angular frequency = $2\varpi f$
- A= amplitude- the distance of the maximum displacement in one period of an oscillation (measured in m)
- k = spring constant (given, measured in N/m)
- L= the length of the string in a simple pendulum
- g= the acceleration due to gravity (on the surface of earth 9.81 m/s^2)
- a= acceleration (measured in m/s)

Important Assumption: No friction and all systems are massless

Simple Pendulum

The first type of simple harmonic oscillator is a simple pendulum:

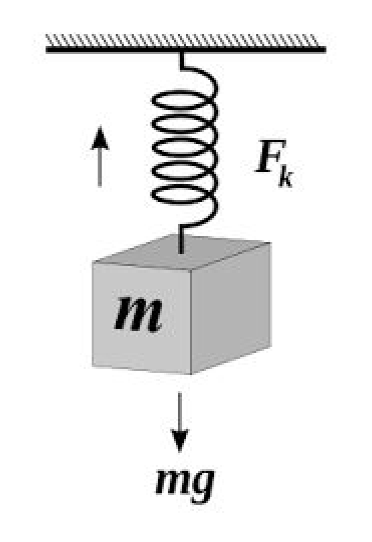
- Restoring Force: Gravity
- Effects
  - Amplitude does not affect Period!
  - Highest velocity and PE at maximum points
  - Highest KE at equilibrium point, a= 0
  - Period is independent of m and A
- $\omega_{sp} = \sqrt{g}/L$



Spring Mass System

The second type of simple harmonic oscillator is a spring mass system:

- Restoring Force: Spring Force
- Effects:
  - Amplitude does not affect Period!
  - Highest velocity and PE at maximum points
  - Highest KE at equilibrium point, a= 0
  - Period is independent of m and A
- $\omega_{ms} = \sqrt{k/m}$



Equations

- The oscillatory motion of simple harmonic systems can be measured using kinematic equations!
  - The same equations from Chapter 1 are used just in different forms to correspond with the variables and situations produced by SHM
- $x = A\cos(\omega t)$
- $v = -\omega A\sin(\omega t) = \sqrt{k/m}(A^2-x^2)$
- $a = -\omega^2 A\cos(\omega t)$
- $v_{max} = -\omega A$
- $PE_s = ½(kx^2)$
- $PE_g = mgh$
- $KE = ½(mv^2)$

Hooke's Law

The force needed to extend or compress a spring by some distance is proportional to that distance

$F_s = -kx$

a= F_s/m

- we are assuming that the spring has neither spring mass nor friction 😊

Common Misconceptions

- Make sure your calculator is in radians
- "ω" is actually a lower case omega not a w
- When the system is at equilibrium, its potential energy is minimum, but kinetic energy is maximum
- The spring mass system always remains the same no matter the gravity of the planet around the spring-mass system, because the only factor that matters is the spring force
- The gravity will affect a simple pendulum
- Each system has a different equation in each situation
- Amplitude does not affect Period!

The Period

The period of a mass spring system can be represented by T.

$T=2\pi\sqrt{(m/k)}$

For a simple pendulum

$T=2\pi\sqrt{(L/g)}$

$T=1/f$

Why SHM?

Simple harmonic motion is a good tool used for modeling oscillating systems.

Examples are:

- AC current
- Foucault pendulum
- Analyzing elastic materials like trampolines

Example Practice Problems

A 5.0 kg mass of a simple pendulum is displaced 20.0 cm from its equilibrium position and released. The string has a length of 25.0 cm. A frequency of 10 Hz is observed. This process takes place on Earth.

a) What is the position of the mass at $t=0.20s$?
b) What is the velocity of the mass at $t=0.20s$?
c) What is the acceleration of the mass at $t=0.20s$?

Practice Problem

Minerva Karelia is just your average woman living on the planet of Zargon in the year 3012. Her son wishes to go to the playground for a playground activity known to Zargonauts only as “Springers”. The system is a spring-mass system on a larger scale which oscillates the child up and down similarly to the way a swing works in a pendulum fashion. Having only lived on the planet for a few years, she does not know how safe the Springers are. So she asks her PRC (Personal Robot Companion) about the safety of the springer. It tells her that the spring constant of the Springer is 57,000 N/m. Her son weighs 205 N. On Zargon, acceleration due to gravity is 8.2 m/s^2. She places her son in the seat of the Springer, and releases him, 4.0 m underneath the equilibrium point. What is the kinetic energy of the system 5.0 seconds after she releases him, when she needs to stop the Springer after she realizes it is not safe?

KE = 30,000J- just slightly unsafe for her child...