Hooke's Law Experiment

Objective: To measure the spring constant of a spring using two different methods.

Background: If a weight, $W = mg$, is hung from one end of an ordinary spring, causing it to stretch a distance x, then an equal and opposite force, F, is created in the spring which opposes the pull of the weight. If W is not so large as to permanently distort the spring, then this force, F, will restore the spring to its original length after the load is removed. The magnitude of this restoring force is directly proportional to the stretch,

$$F = -kx$$

The constant k is called the spring constant. To emphasize that x refers to the change in length of the spring we write

$$F = mg = -k\,\Delta \ell \tag{1}$$

In this form it is apparent that if a plot of F as a function of Δl has a linear portion, this provides confirmation that the spring follows Hooke's Law and enables us to find k.

An additional approach is possible. One definition of simple harmonic motion is that it is motion under a linear, "Hooke's Law" restoring force. Note that for simple harmonic motion, the period does not depend upon the amplitude of the oscillation. For such a motion, we have

$$T^2 = 4\pi^2 m / k \tag{2}$$

where k again is the spring constant, T is the period of the pendulum and m is the mass that is oscillating. Thus, the mass includes the mass of the spring itself. However, the entire spring does not vibrate with the same amplitude as the load (the attached mass) and therefore it is reasonable to assume that the effective load (m) is the mass hung from the end of the spring plus some fraction of the mass of the spring. Based on similar experiments, one third of the mass of the spring is a good estimation of the effective load due to the spring, thus

$$m = m_{load} + m_{es} = m_{load} + \frac{1}{3} m_{spring}$$

where m_{es} is the effective load of the spring. Using this in Eq. (2), we find

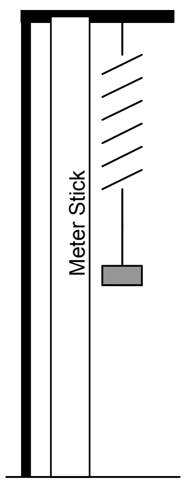
$$k = \frac{4\pi^2 \{m_{load} + 1/3(m_{spring})\}}{T^2} \tag{3}$$

The effective load of the spring can be determined for a particular spring using the following process. The equation for T^2 can be written in terms of m_{load} and m_{es} ; m_{es} can then be determined from a graph of T^2 versus m_{load}. Note that this assumes that m_{es} is constant.

Eq (3) uses an approximation for the contribution of the mass of the spring to the oscillation. If we rewrite Eq (2) as the effective mass of the spring and hanging mass (load), then

$$T^2 = 4\pi^2 (m_{load} + m_{ES}) / k = \frac{4\pi^2}{k} m_{load} + \frac{4\pi^2 m_{ES}}{k} \tag{4}$$

where m_{load} is the hanging mass and m_{ES} is the effective mass of the spring. If we assume that the effective spring mass is the same for all loads, then a graph of period squared (T^2) vs. hanging mass (m_{load}) is a straight line, where $4\pi^2/k$ is the slope and $4\pi^2 m_{ES}/k$ is the intercept.



Procedure:

Part 1

1. Hang a spring from a horizontal metal rod.
2. Attach a mass hanger directly to the bottom of the hanging spring and record the position of the bottom of the mass hanger relative to a meter stick.
3. Add masses to the spring and record the position of the bottom of the mass hanger.

Part 2

1. Hang a mass from the spring and use a stopwatch to time 15 oscillations of the mass and spring.
2. Repeat for other masses.

Assemble report using a staple in the upper left corner.

Physics 1408   Section A1

HOOKE'S LAW AND A SIMPLE SPRING

Your Name

A lab report should include a title page like this one, with all of the appropriate information.

Partner(s): Full Name(s)

Date Performed: October 21, 2005

TA: Full Name

Abstract

Two experiments were performed to find the spring constant of a steel spring. The spring constant was determined statically, by measuring its elongation when subjected to loading, and dynamically, by measuring the period of a mass hung from one end and set into vertical oscillation. The resulting values of 2.94 ± 0.01 N/m and 2.98 ± 0.02 N/m, respectively. Our spring's behavior followed Hooke's law to within the limits of accuracy of the two experiments.   (76 words)

An Alternate Abstract:
The purpose of this experiment was to measure and compare the spring constant of a steel spring using two different procedures. First we investigated the relationship between the force applied to a spring and the displacement of the spring from its rest length. We hung various masses from the springs, and measured the vertical displacement. We found a spring constant of 2.94 ± 0.01 N/m. Our results confirmed Hooke's Law, $F_s = -kx$. In the second procedure, we set the spring into vertical oscillation with a suspended mass and measured the period of oscillation. Using this method, we found a spring constant of 2.98 ± 0.02 N/m. Our results verified that the period of oscillation depended on the effective mass of the spring and the period of oscillation. (128 words)

A Poor Abstract – Too long because it has too much detail and unnecessary information. ***(The worst problems are in italics.)***

The purpose of this experiment was to determine the spring constant k of a steel spring using two different methods. First we investigated the relationship between the force applied to a spring and the displacement of the spring from its rest length in order to verify Hooke's law. *We hung masses of 0.01 kg, 0.20 kg, 0.30 kg, 0.04 kg, 0.05 kg, 0.06 kg, 0.70 kg, and 0.80 kg from the springs, and recorded the vertical displacements. We made four measurements for each mass hung from the spring and used the average of the four values in order to reduce random error. In this method, the main cause of error was measurement.* We found a spring constant of $k = 2.94 \pm 0.01$ N/m. Our results confirmed Hooke's Law, *the well known relationship that the magnitude of an elastic restoring force on a spring is directly proportional to the displacement of the spring. This relationship is named after the 17th century scientist Hooke who studied it.* Next we measured the period of a mass hung from one end of a spring and set into vertical oscillation. *We performed this process using the four different masses 0.145 kg, 0.105 kg, 0.055 kg, and 0.025kg. The period of each mass was measured three times using three different amplitudes of oscillation.* We found that the spring constant depended on the effective mass of the spring and the period of oscillation. The period of the motion was the same whether the amplitude of the oscillation is large or small. *In this method, the main cause of error was reaction time.* Using this method we found a spring constant of 2.98 ± 0.02 N/m. This value is consistent with the result obtained using the first method. (291 words)

Remember to be concise.

Name: ____*Your Name*________ Date:__Date Exp. Performed

Partner: ___*Partners Full Name*__

TA's Initials on data sheet

Hooke's Law and a Simple Spring

You can include uncertainty for a tool such as a meter stick in the top of a data table.

Part 1

Uncertainty can be written as a percentage.

Table 1

Position	Mass (g) ±1%	Location of the Mass Hanger Reference in cm ±0.05cm			
		Trial 1	Trial2	Trial 3	Trial 4
Reference	0	69.55	69.50	69.50	69.50
1	1	69.27	69.19	69.18	69.17
2	3	68.61	68.50	68.53	68.52
3	5	67.95	67.87	67.88	67.86
4	10	66.42	66.20	66.21	66.20
5	20	62.90	62.89	62.90	62.93
6	40	56.32	56.22	56.30	56.23
7	60	49.65	49.60	49.61	49.6
8	80	42.97	42.97	42.95	42.95
9	100	36.32	36.30	36.32	36.32
10	120	29.63	29.70	29.72	29.72
11	140	23.07	23.05	23.10	23.12

If the value is four sig figs then include the trailing 0.

If you drop that zero then it is 3 sig figs! This should have been 49.60!

For the subtraction all the uncertainties were the same; thus it was put in the top of a column; that was not possible for the average.

Do you know why there are only 2 sig figs here?

Table 2

Force (N) ±1%	Displacement (x10^{-2}m)					Spring Constant (N/m)
	Trial 1 ±0.07	Trial 2 ±0.07	Trial 3 ±0.07	Trial 4 ±0.07	Average	
0.00981	-0.28	-0.36	-0.37	-0.38	-0.35 ± 0.02	2.8 ± 0.2
0.0294	-0.94	-1.05	-1.02	-1.03	-1.01± 0.02	2.91± 0.08
0.0491	-1.60	-1.68	-1.67	-1.69	-1.66± 0.02	2.96± 0.05
0.0981	-3.13	-3.35	-3.34	-3.35	-3.29± 0.05	2.98± 0.05
0.196	-6.65	-6.66	-6.65	-6.62	-6.65± 0.01	2.95± 0.03
0.392	-13.23	-13.33	-13.25	-13.32	-13.28± 0.02	2.95± 0.03
0.589	-19.90	-19.95	-19.94	-19.95	-19.94± 0.01	2.95± 0.03
0.785	-26.58	-26.58	-26.60	-26.60	-26.59± 0.01	2.95± 0.03
0.981	-33.23	-33.25	-33.23	-33.23	-33.24± 0.01	2.95± 0.03
1.18	-39.92	-39.85	-39.83	-39.83	-39.86± 0.02	2.95± 0.03
1.37	-46.48	-46.50	-46.45	-46.43	-46.47± 0.02	2.96± 0.03

Average Spring Constant k = __2.94 ± 0.01 N/m_

Spring Constant k from Graph = __2.95 N/m _________

Calculate the standard error.

There is a way to find the uncertainty for slopes of graphs, but we will not do that in this course.

Remember to include units and uncertainty.

Name: ____*Your Name*_________

Part 2:

Mass of spring = _*10.19 ± 0.02 x 10^{-3} kg*_____

Cross out mistakes with a single line; do not use white-out.

With 1% uncertainty in the slotted masses, the uncertainty for 145g is ±1g and for 55g, it is ±0.06g; thus for the smallest two loads, can be written in the form below.

Table 3

Load (x10^{-3} kg) *(mass of spring) ±1%*		*145*	*105*	*55.0*	*25.0*
Time for 20 oscillations (s) ±0.08	Trial 1 (small)	*27.94*	*23.98*	*17.60*	*12.32*
	Trial 2 (medium)	*27.79*	*24.06*	*17.44*	~~*12.56*~~ *12.40*
	Trial 3 (large)	*27.90*	*23.95*	*17.34*	~~*12.03*~~ *12.34*
Average 20 oscillations (s)		*27.88 ± 0.04*	*24.00± 0.03*	*17.46± 0.08*	*12.35± 0.02*
Period (s)		*1.394± 0.002*	*1.200± 0.002*	*0.873± 0.004*	*0.618± 0.001*
Period ^{2} (s^2)		*1.943 ± 0.006*	*1.440 ± 0.004*	*0.762± 0.007*	*0.382 ± 0.001*
k from Eq. (3) (N/m)		*3.01 ± 0.03*	*2.97± 0.03*	*3.02± 0.04*	*2.94± 0.03*

Do not cross out mistakes this way! Sometimes what you had was correct. You may want to be able to read it.

Average Spring Constant k = ___*2.98± 0.02 N/m*_

Period was 12.35s/20= 0.6175s Uncertainty was 0.001; thus we write the period with 3 sig figs (0.618)

Spring Constant k from Graph = ___*3.01 N/m*_

Spring's Effective Mass from Graph = __*0.0040 kg*

% Difference between k in Part 1 and Part 2 = __*2 %*_

k in parts 1 and 2 was 3 sig figs; but the difference between the two values is one sig fig!!!!

Remember to include an uncertainty!

Guessing and then seeing what the value is for the average of a small number of trials helps build your ability to predict uncertainty.

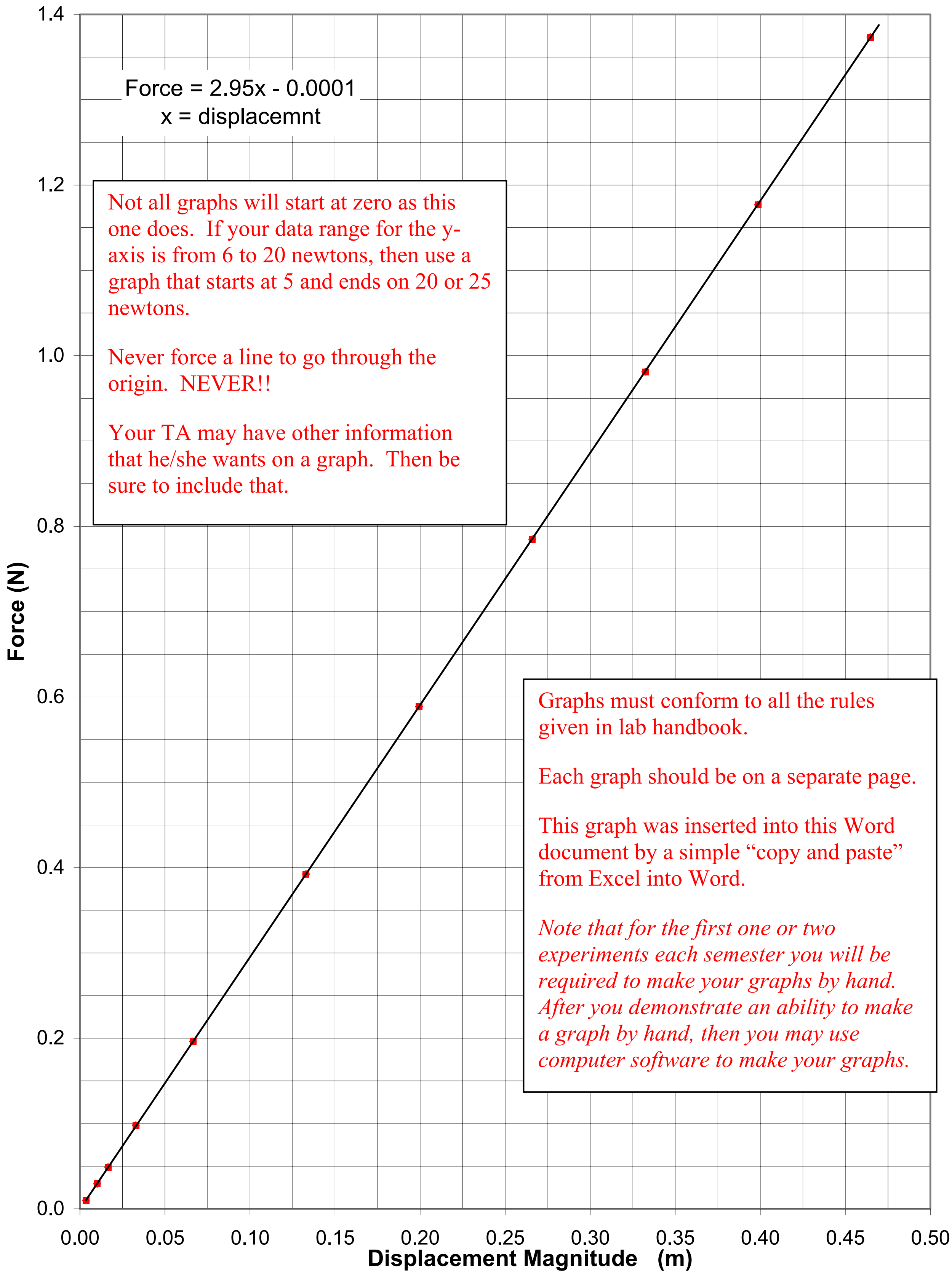
Why 0.08 in this case? One way to measure your reaction time is to have a friend hold a meter stick vertically between your thumb and first finger; note the cm mark between your fingers; the friend drops the meter stick, and you catch it. Determine how far it fell before your caught it. Since it had no initial velocity and the only force acting on the meter stick is gravity, then $d=\frac{1}{2}gt^2$. For most people, the distance the meter stick falls is about 15 cm. However, there is a difference between someone dropping a meter stick and timing an oscillating object. When timing an object, we can observe the motion and use the rhythm to reduce reaction time error. This might reduce the timing error to a third of the original value: $t_{reaction\ time}/3 = \text{sqrt}(2d/g)/3 = 0.06$ s

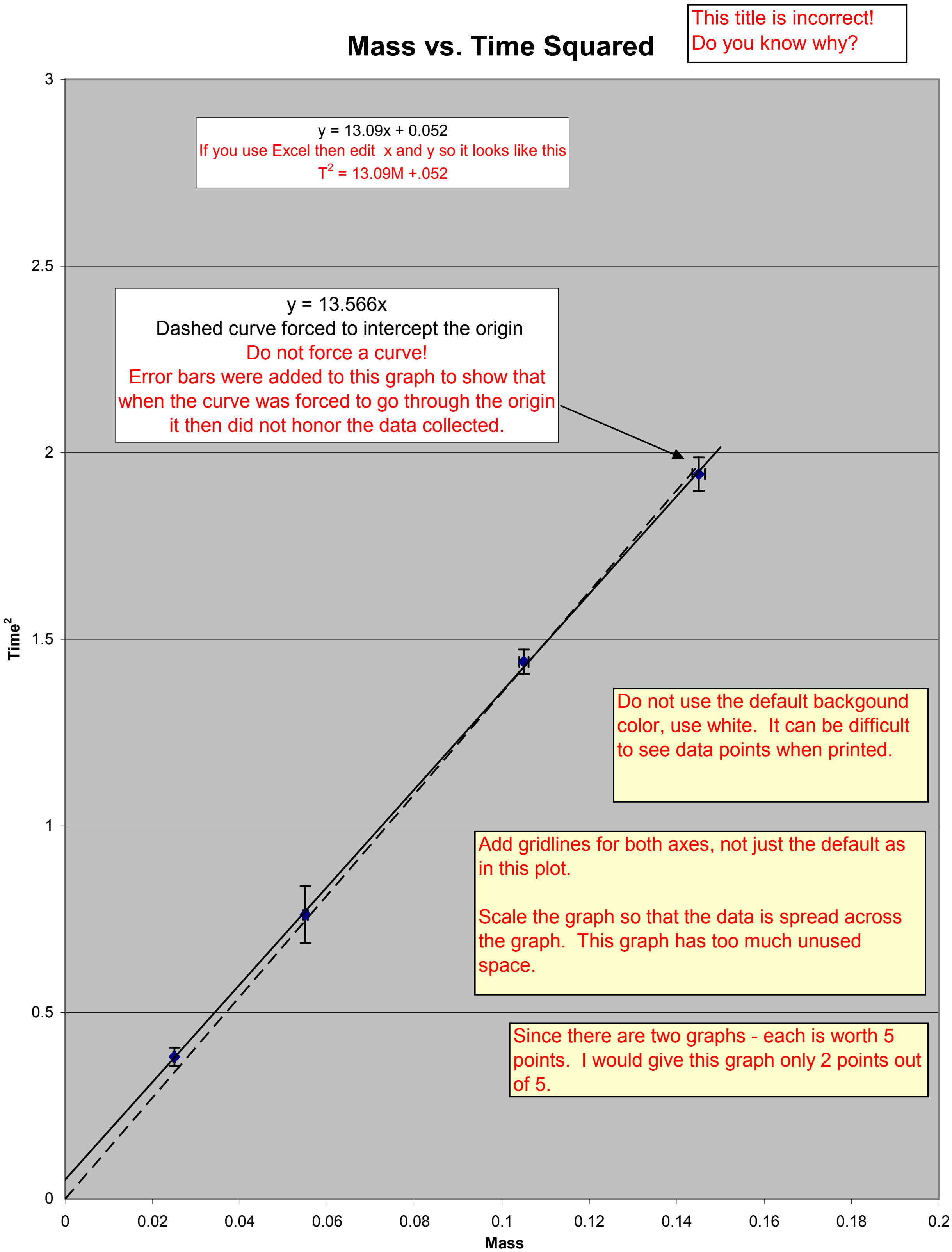
However, there are two uncertainties – starting the stopwatch and stopping the stopwatch; thus, we need to propagate the error. $\Delta t = \text{sqrt}(\Delta t_{start} + \Delta t_{start}) = \Delta t_{reaction\ time}\ \text{sqrt}(2) = 0.08$s

We might have made a different assumption, then we might have used had a value or 0.05 to 1.0 s.

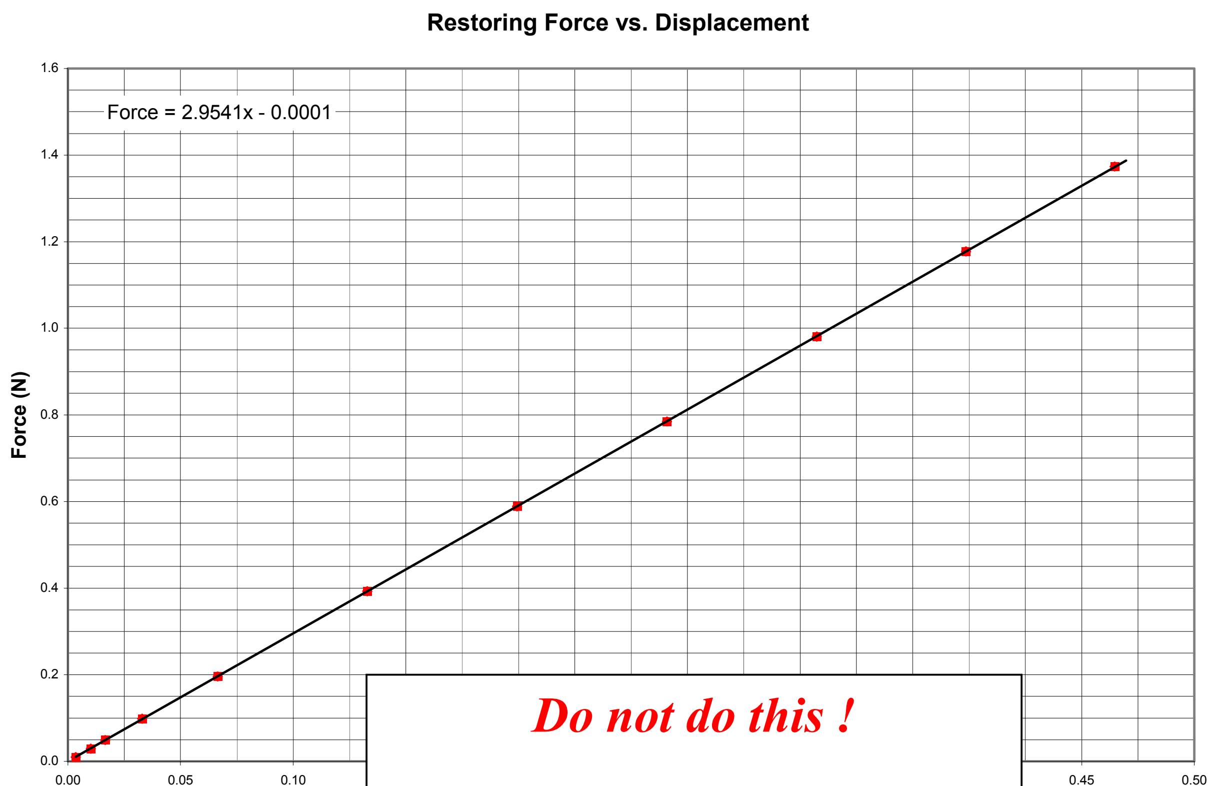
Remember to include units and uncertainty

Restoring Force vs. Displacement Magnitude



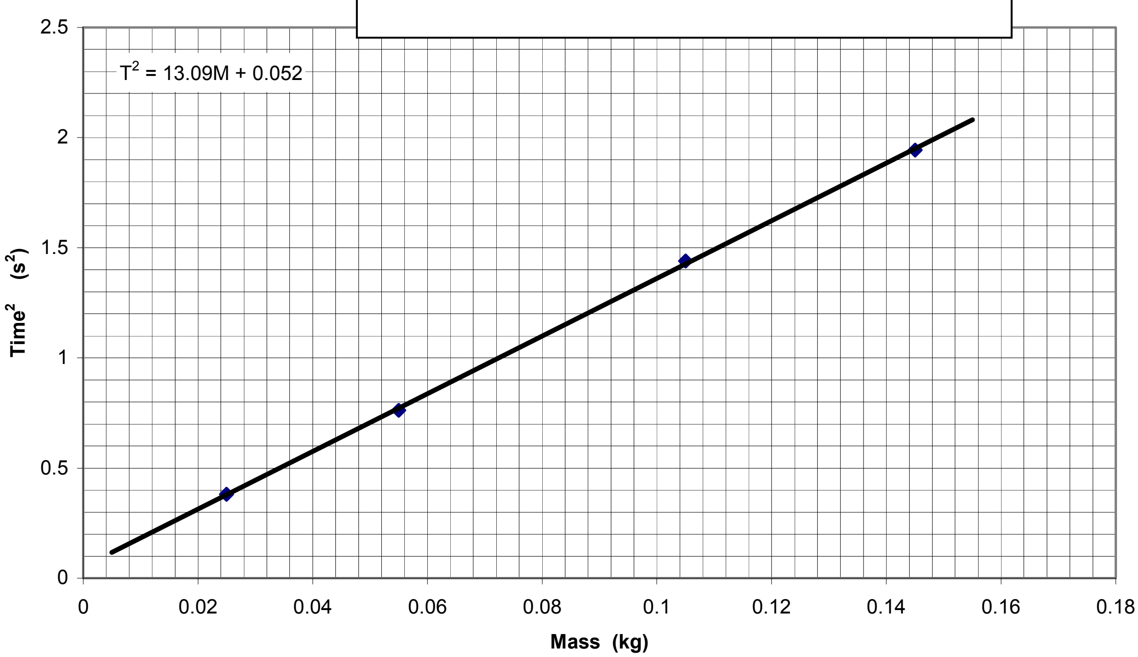


Mass vs. Time Squared
This title is incorrect! Do you know why?
y = 13.09x + 0.052
If you use Excel then edit x and y so it looks like this
T2 = 13.09M +.052
y = 13.566x
Dashed curve forced to intercept the origin
Do not force a curve!
Error bars were added to this graph to show that when the curve was forced to go through the origin it then did not honor the data collected.
Do not use the default backgound color, use white. It can be difficult to see data points when printed.
Add gridlines for both axes, not just the default as in this plot.
Scale the graph so that the data is spread across the graph. This graph has too much unused space.
Since there are two graphs - each is worth 5 points. I would give this graph only 2 points out of 5.
Time2
Mass
0
0.5
1
1.5
2
2.5
3
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2



Do not do this !

One graph per page. Not two or more per page and not embedded in text.



Sample Calculations

1. Displacement: the length that the spring is stretched

x = Displacement = Location with Mass 1 (0.010kg) - Reference Location

$$x = 66.42\times10^{-2}\,m - 69.55\times10^{-2}\,m = -3.13\times10^{-2}\,m$$

2. Uncertainty of displacement ($\Delta\ell$): Propagation of error for addition and subtraction

$$\Delta x = \sqrt{(\text{uncertainty in reference})^2 + (\text{uncertainty in location 1})^2}$$

$$\Delta x = \sqrt{(0.05\times10^{-2}\text{m})^2 + (0.05\times10^{-2}\text{m})^2}$$

$$\Delta x = 0.07\times10^{-2}\text{m}$$

3. Force on spring from the hanging mass

$$F = mg$$

$$F = (10.0g)(1kg/1000g)(9.81m/s^2) = 0.0981N$$

4. Standard Error for Average Displacement for 0.9811N force

$$\text{Standard Error} = \sqrt{\frac{\sum(x_i - \bar{x})^2}{(N-1)N}}$$

$$\Delta x = \{[(-3.13\times10^{-2}\text{m} - (-3.29\times10^{-2}\text{m}))^2 + (-3.35\times10^{-2}\text{m} - (-3.29\times10^{-2}\text{m}))^2$$
$$+ (-3.34\times10^{-2}\text{m} - (-3.29\times10^{-2}\text{m}))^2 + (-3.35\times10^{-2}\text{m} - (-3.29\times10^{-2}\text{m}))^2]/[(4-1)4]\}^{1/2}$$

$$\Delta x = 0.05\times10^{-2}\ \text{m}$$

5. Using Hooke's Law (F = - kx) to find the spring constant

$$k = -F/x$$

$$k = -0.0981/(-3.29\times10^{-2}) = 2.98$$

No units! No credit would be given for this sample calculation.

6. Spring constant uncertainty: Propagation of error for multiplication and division

$$\Delta k = k\sqrt{(\Delta F/F)^2 + (\Delta x/x)^2}$$

$$\Delta k = 2.98\text{N/m}\sqrt{\left(\frac{0.01\times0.0981N}{0.0981N}\right)^2 + \left(\frac{0.05\times10^{-2}\,m}{3.29\times10^{-2}\,m}\right)^2} = 0.05\ \text{N/m}$$

7. Spring constant from period of oscillation

$$k = \frac{4\pi^2\{m_{load} + 1/3(m_{spring})\}}{T^2}$$

$$k = 4(3.14)^2(145\times10^{-3}\,kg + \frac{1}{3}\left(10.19\times10^{-3}\,kg\right))/\left(1.392s\right)^2$$

$$k = 3.01\ \text{N/m}$$

8. Spring constant uncertainty: propagation of error for T^2

$\Delta A = nA(\Delta T/T)$ where $A = T^n$ $n = 2$

$\Delta T = 2(1.394s)^2(.002s/1.394s) = 0.006\ s^2$

9. Spring constant from the slope from T^2 vs. M_{load} graph

$$slope = \frac{(2\pi)^2}{k} \qquad k = \frac{(2\pi)^2}{slope}$$

$$k = \frac{(2 \times 3.14)^2}{13.09\,s^2/kg} = 3.01 N/m$$

10. Spring's Effective Mass, $M_{ES,}$ from the intercept

$$Intercept = \frac{(2\pi)^2 M_{ES}}{k} \qquad M_{ES} = \left(k \times Intercept\right)/(2\pi)^2$$

$$M_{ES} = \left(3.01\text{N/m} \times 0.052 s^2\right)/(2 \times 3.14)^2 = 0.0040\ kg$$

11. % difference for spring constant k

$$\%\ difference = \frac{\left|\text{Measured Value1 - Measured Value 2}\right|}{\left|\text{Measured Value1+Measured Value 2}\right|/2} \times 100\%$$

$$\%\ difference = \frac{|\ 2.94\text{N/m} - 2.98\text{N/m}\ |}{|\ 2.94\text{N/m} + 2.98\text{N/m}\ |/2} \times 100\% = 1\%$$

Discussion

(One to three sentences are enough to introduce what was done. The procedure is in the lab manual. Do not rewrite the procedure. More than that is wasting your time and the lab instructor's time.) In part 1, a spring was hung vertically with a mass hanger attached to the lower end of the spring, and masses from 1g to 140g were added. The downward location of the spring was measured once it came to rest. *(A succinct explanation of the physics principle used in the experiment.)* In this configuration, two equal and opposite forces acted on the hanging mass: gravity directed downward and the spring's elastic restoring force directed upward, in the opposite direction of displacement. Using Hooke's Law ($F = -kx$), a spring constant was calculated for each measurement. *(How the result demonstrated a physics principle.)* The spring constants for each value of displacement are the same, within experimental uncertainty (Table 2), which verifies Hooke's law. *(Only the important result is provided. Not a list of each and every number on the data sheet. Note that final numerical values include an estimate of uncertainty.)* The average spring constant is 2.94±0.01 N/m.

(Analysis of graph: shape of curve, for a straight line, the meaning of slope and intercept for your graph.) A graph of force versus the magnitude of displacement resulted in the expected straight line in the range of forces examined and is consistent with Hooke's law. The slope of this line, 2.95 N/m, is the spring constant, which agrees with value found by taking the average of the calculated spring constant (2.94±0.01 N/m). *(You do not have to explain how they agree if you show the numbers or refer to a Table; but do not write that values agree without some reference.)* The intercept for the best fit straight line intersects close to the origin, which is also consistent with Hooke's law.

(Sources of error are offered that are consistent with the experimental results.) The sources of error in this part of the experiment are due to the precision of the location measurement using the meter stick and the accuracy of the slotted masses. The meter stick was mounted vertically and behind the spring. The location was measured relative to the base of the mass hanger. Effort was made to sight the measurements directly; however, because of the location of the meter stick it was necessary to view the meter stick at a slight angle. However, this sighting was required for each measurement, and the displacement was the difference between the location and the reference. Thus, this systematic error due to parallax should be minimal. However the random error of measurement precision remains. For displacements 20 cm or more, the uncertainty of the displacement of the spring is 0.5 % or less and has little impact on the uncertainty of k; in those cases the 1% uncertainty in the slotted masses has the greatest contribution to the uncertainty of k. However, for small displacements the displacement uncertainty has the largest impact on the uncertainty in k. For example, the 1.0 g mass displaced the spring by -0.0035 ± 0.0002 m, a relative uncertainty of 6%. *(You may offer a suggestion for improving the experiment, but it must focus on the most prominent error and be consistent with the sources of errors. This is not a place to "trash" the experiment.)* Using a motion sensor to measure distance would increase the precision for small displacements.

(A brief introduction to part 2.) In Part 2, we determined k dynamically using the period of an oscillating mass. The time for twenty oscillations was measured for five different masses; for each mass the period of oscillation was measured three times using different oscillation amplitudes, as suggested by out lab instructor. *(A succinct explanation of the physics*

principle used in the experiment.) The period of a mass oscillating vertically on a spring depends on the spring constant and the mass of the oscillating object, but not on the amplitude of the oscillation. *(How the result demonstrated a physics principle.)* Our measurements confirmed that the amplitude of oscillation, within experimental uncertainty, did not affect period (Table 3),

To reduce the reaction time, we observed the motion and used the rhythm to start and stop the stopwatch. (*How the independent variables affected the dependent variables.)* For small masses, the period of the oscillation is shorter; this is consistent with Eq (2). These shorter periods for the 55g and 25g masses made accuracy in the timing both critical and difficult. The measured times for 20 oscillations of the 55g mass are not as consistent as for the other masses. This was the result of reaction time random error. Two measurements for 20 oscillations of the 25g mass were so different from the other measurements that we made additional measurements and replaced those data points. There was another complication for these smaller mass, large amplitude oscillations caused the slotted masses to bounce on the mass hanger. This meant that we had to use smaller amplitude differences between the large and small amplitude oscillations for the smaller masses.

(You may need to combine two equations.) Using Eq (3), we found k for the four different loads added to the spring. The four values of k for the four different masses were in agreement (Table 3). The average value of k is 2.98±0.02 N/m. (*Always include units; 2.98± 0.02 without units would be meaningless.)*

(Analysis of graph.) A graph of T^2 vs. M_{load} is a straight line and consistent with the theory that the period is a function of the effective mass of the spring and the spring constant of the spring, Eq, (4). *(Important results.)* The spring constant k from the slope is 3.01 N/m; the effective mass of the spring M_{ES} from the intercept of the best fit line is 4.0g, which is approximately 40% of the mass of the spring, which is somewhat higher than the fraction used in Eq. (3).

The sources of error in this part of the experiment are due to the accuracy of the slotted weights and the accuracy of the time measurements. *(There is no need to repeat what you have already discussed.)* As mentioned previously, the reaction time uncertainty is greater for the smaller loads. However, due to the care that taken in the time measurements and the fact that 20 different oscillations were measured, the uncertainty in the time measurements was not as important in this experiment as the uncertainty in the slotted masses. There is uncertainty (1%) in the mass of the slotted weights. It would have been prudent to have measured the masses on the triple beam balance so that we would have less uncertainty in the mass of the oscillating weights; however, we did not make those measurements.

The value of the spring constant found in Part 1 (2.94±0.01 N/m) and Part 2 (2.98± 0.02 N/m) do not agree. *(Discuss how results agree using either uncertainty and/or percent differences.)*However, the percent difference between the two values is only 1%. One possible explanation for the small discrepancy may be that the time measurements were precise, but not accurate due to a systematic error in the timing. If our time measurements of the twenty oscillations were low by as little as 0.05s, then the spring constant values would agree.

(A brief conclusion.) Besides measuring the spring constant using two very different methods, we verified Hooke's law, verified the linear relationship between period squared and load for a vertically oscillating spring, and observed that the amplitude of the oscillations did not affect the period.

Answers to Questions (if any): Answer questions in complete, grammatically correct sentences.