Circular Motion & Rotational Mechanics

•••

Semester 2 Review Project - Sonya Kalara, Ellie Kim, and Charlotte Spry

Definitions

I. Uniform Circular Motion - an object that moves in a circle at a constant velocity
   A. Magnitude of the velocity stays constant, but direction is constantly changing
II. Rotation - body turns about an internal axis
III. Revolution - when a body turns on an external axis
IV. Types of Behavior
   A. Radial - Behavior towards and away from the center of the circle
   B. Tangential - behavior along the edge of the circle
      1. In order to turn an object, an unbalanced, external force must be applied towards the center of the circle
V. Centripetal - “center seeking” forces
   A. Causes a centrifugal force, which is a faux “center fleeing” force that appears when the reference frame is accelerating

Centripetal Force

I. 3 Quantities to determine the magnitude of the force necessary to retain a circular path
   A. Mass
      1. Larger Mass = More Force Necessary
   B. Velocity
      1. Higher Velocity = More Force necessary
   C. Radius
      1. Larger Radius = Lower Force necessary

II. EQUATION
   A. $F_c = ma_c$
   B. $a_c = v^2 / r$
   C. $F_c = mv^2 / r$

Frequency & Period

1. Frequency - # of revolutions per second (f)
   a. Hertz (Hz) = 1 revolution / second
2. Period - Time to take one revolution (T)
   a. Seconds
   b. $T = 1 / f$
   c. $2\pi r / T = v$

Using Rotational Motion

I. Banking Curves
   A. Reduces the chance of skidding because the normal force has a component towards the center, acting as a centripetal force, which reduces the reliance on friction
   B. For any angle theta, there is a speed where friction isn't necessary, called "design speed"
      1. $F_N \sin(\theta) = mv^2 / r$

Circular Motion & Angular Quantities

I. Centripetal Acceleration in terms of angular velocity
   A. $\alpha_c = \omega^2 r$
II. Frequency in terms of angular velocity
   A. $f2\pi = \omega$
III. KINEMATIC EQUATIONS - Work when acceleration is constant
   A. $\omega_f = \omega_i + \alpha\Delta\theta t$
   B. $\Delta\theta = \omega_i \Delta t + (½)\alpha\Delta t^2$
   C. $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Torque & Moment of Inertia

I. Torque - The “twisting force” which causes rotation
   A. $\tau = F\square r * \sin\Theta$
II. Rotational Inertia
   A. $a = \Sigma F/m$ [linear]
   B. $\boldsymbol{\alpha} = \Sigma\boldsymbol{\tau}/I$
III. Moment of Inertia - measure of a body’s resistance to rotational motion
   A. Newton’s 3rd Law
   B. $\tau = \boldsymbol{\alpha} I$
   C. $I = \Sigma mr^2$

Angular Momentum & Rotational Kinetic Energy

I. Rotational Kinetic Energy
   A. Translational - $\frac{1}{2} mv^2$
   B. Rotational - $\frac{1}{2} Iw^2$
II. Angular Momentum
   A. Linear Momentum - $p = mv$
   B. Angular - $L = Iw$
      1. $\triangle\tau = \triangle L / \triangle t$

Angular Quantities versus Linear Quantities

Quantity	Linear	Angular	Relationship
Position	Distance/Length (d/l) in m	Angle (θ) in rad	$\theta = l / r$
Velocity	Velocity (v) in m/s	Omega (ω) in rad/s	$\omega = v / r = \Delta\theta / \Delta t$
Acceleration	Acceleration (a) in m/s^2	Alpha (α) rad/s^2	$\alpha = a / r = \Delta\theta / \Delta t$

Note: 2πrad = 360° → 2π (180/π) = 360°

Common Misconceptions and Mistakes

1. Forgetting to convert between diameter and radius
2. Centrifugal force: fictitious outward force experienced because your reference frame is accelerating
   a. When car makes a sharp left, the direction of the net force acting on your body is left, but the your body feels like it's being pushed to the right because of inertia
3. Remember to convert between angular and linear quantities, and remember the specific functions of each
4. Centripetal Force is never random; it is always a net force that is made up of other forces. The phrase "centripetal force" is simply a label for an already existing, center seeking force
   a. Normal force on a ferris wheel, tension for a ball on a string, friction for a car turning, gravity for planets

Walkthrough Problem

A manufacturer of CD-ROM drives claims that the player can spin the disc as frequently as 1200 revolutions per minute.

a. If spinning at this rate, what is the speed of the outer row of data on the disc; this row is located 5.6 cm from the center of the disc?

b. What is the acceleration of the outer row of data?

a. *Ans. 7 m/s*

b. *Ans.* $8.8 \times 10^2\ m/s^2$

Practice Problem #1

A loop de loop track is built for a 938-kg car. It is a completely circular loop - 14.2 m tall at its highest point. The driver successfully completes the loop with an entry speed (at the bottom) of 22.1 m/s.

a. Using energy conservation, determine the speed of the car at the top of the loop.

b. Determine the acceleration of the car at the top of the loop.

c. Determine the normal force acting upon the car at the top of the loop.

a. *Ans. 14.5 m/s*

b. *Ans. 30. m/s^2*

c. *Ans. 1.9 x 10^4 N*

Practice Problem #2

A popular yo-yo trick is to have the yo-yo "climb" the string. A yo-yo with mass .5 kg and moment of inertia of .01 begins by rotating at an angular velocity of 10 rad/s. It then climbs the string until the rotation of the yo-yo stops completely. How high does the yo-yo get?

Ans. 0.102 m