

Projectile Motion

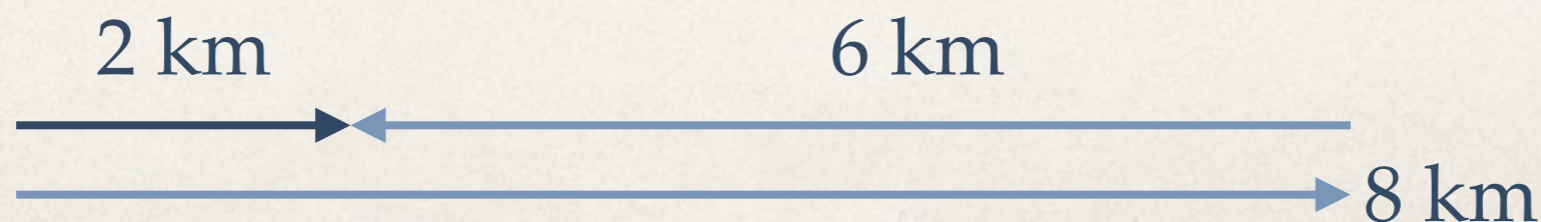
General Physics

Vectors

- ❖ Often helpful to draw out vectors when depicting physics situations
- ❖ Arrows show direction
- ❖ Length of the arrow represents relative magnitude

Adding Vectors

- ❖ In one dimension, adding vectors is a simple matter of adding or subtracting values
- ❖ Sven the Irresponsible walks 8 km east, realizes he left his kid at the ice cream stand, then walks 6 km west to retrieve her
 - ❖ The scalar distance traveled is $8 \text{ km} + 6 \text{ km} = 14 \text{ km}$
 - ❖ The *net* or *resultant* displacement accounts for the change in direction. $8 \text{ km} - 6 \text{ km} = 2 \text{ km east}$

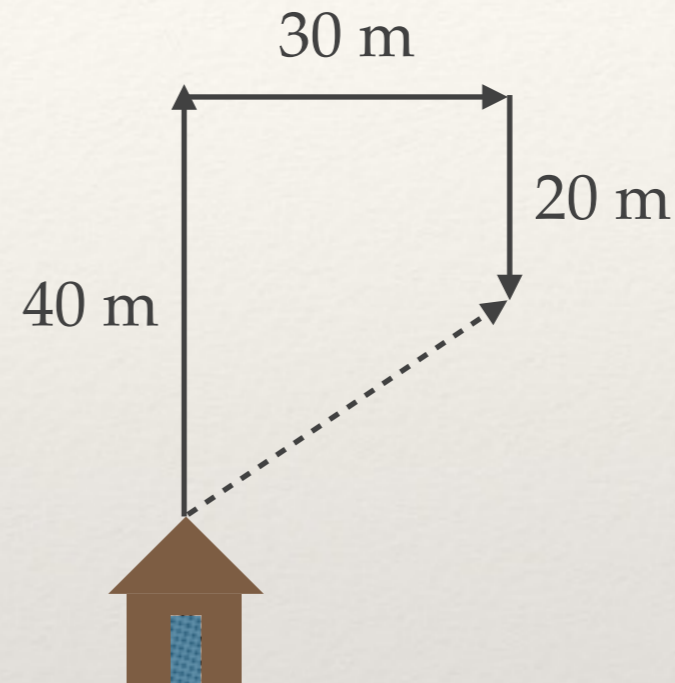


Adding Vectors

- ❖ However, simple arithmetic cannot be used if the two vectors are not along the same line

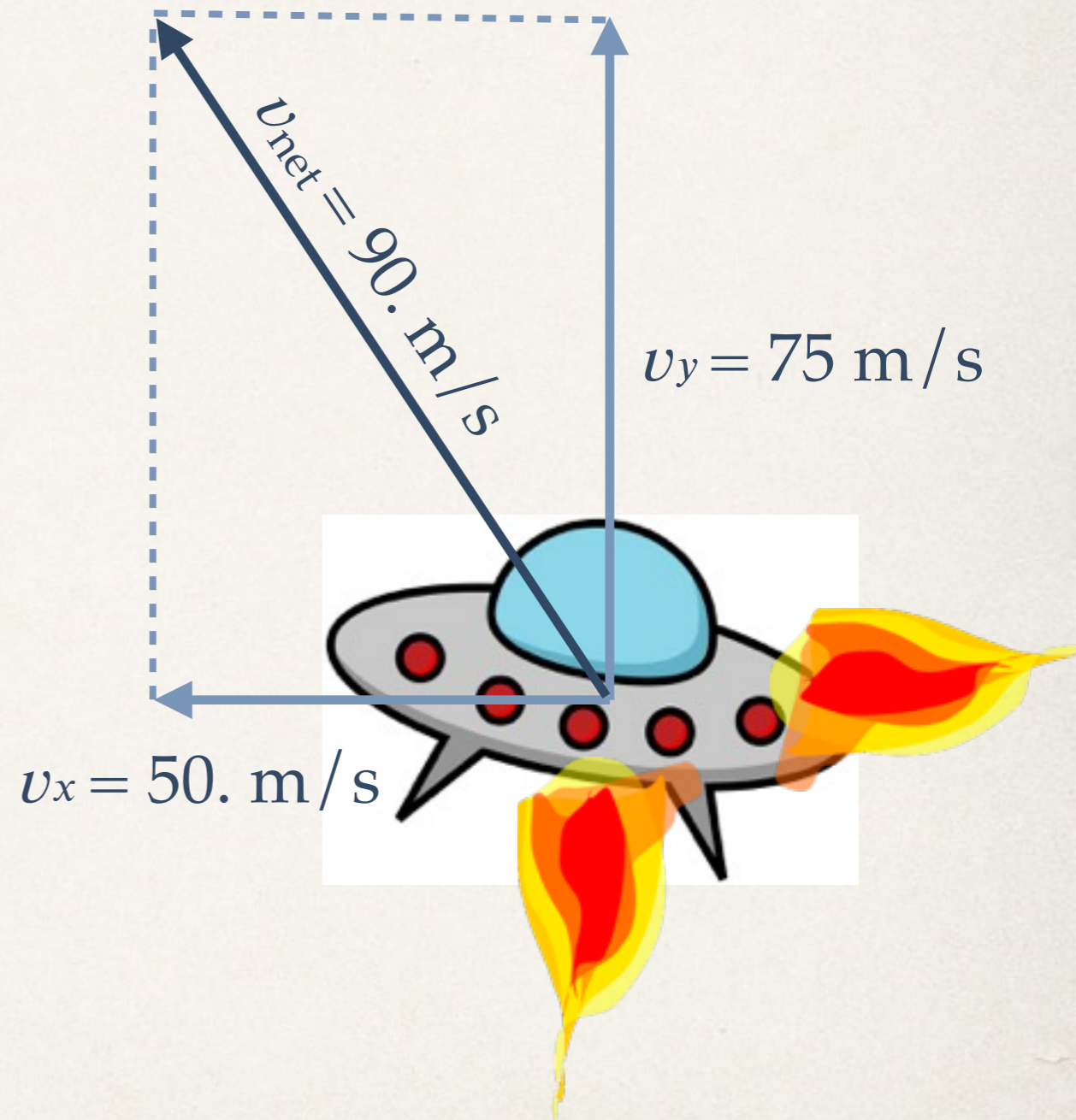
Distance vs. Displacement

- ❖ Little Sally leaves her home, walks 40 m north, 30 m east, and 20 m south
- ❖ How far is Sally from home?
- ❖ $\sqrt{(20^2 + 30^2)} = 36 \text{ m}$
- ❖ In what direction?
- ❖ $\tan^{-1}(20/30) = 34^\circ \text{ North of East}$
- ❖ **36 m @ 34° N of E**
- ❖ This is Sally's *displacement*



Adding Vectors

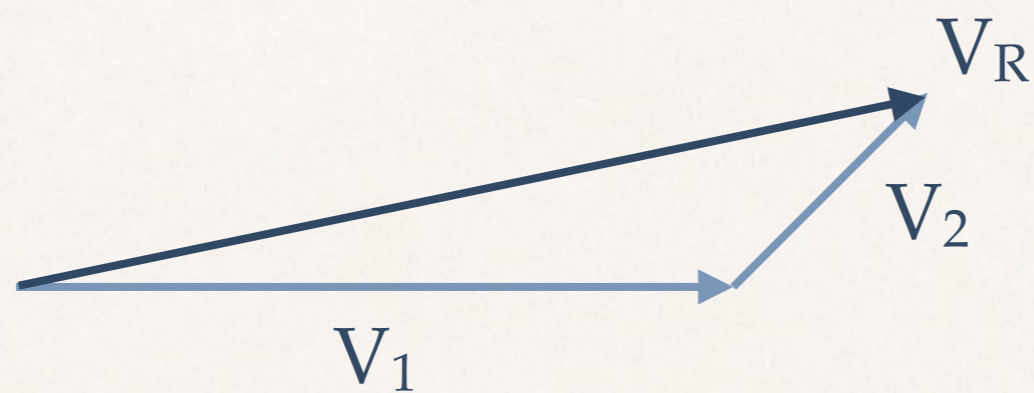
- ❖ The same idea applies to all vectors
- ❖ An alien spaceship cranks on two thrusters, one launching it to the left at $50. \text{ m/s}$, the other propelling it up at 75 m/s .
- ❖ What is the *net* velocity of the spaceship?
- ❖ $v_{\text{net}} = \sqrt{v_x^2 + v_y^2}$



Methods of Adding Vectors

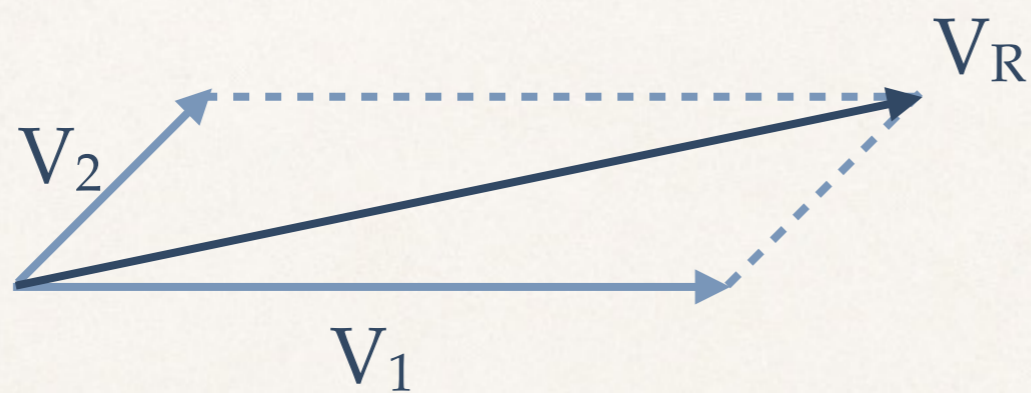


Methods of Adding Vectors



Tail-to-Tip Method

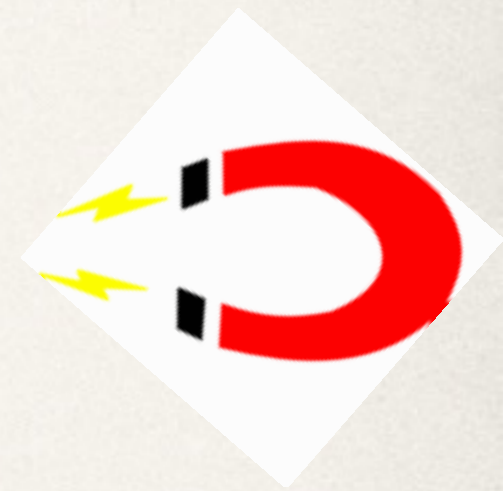
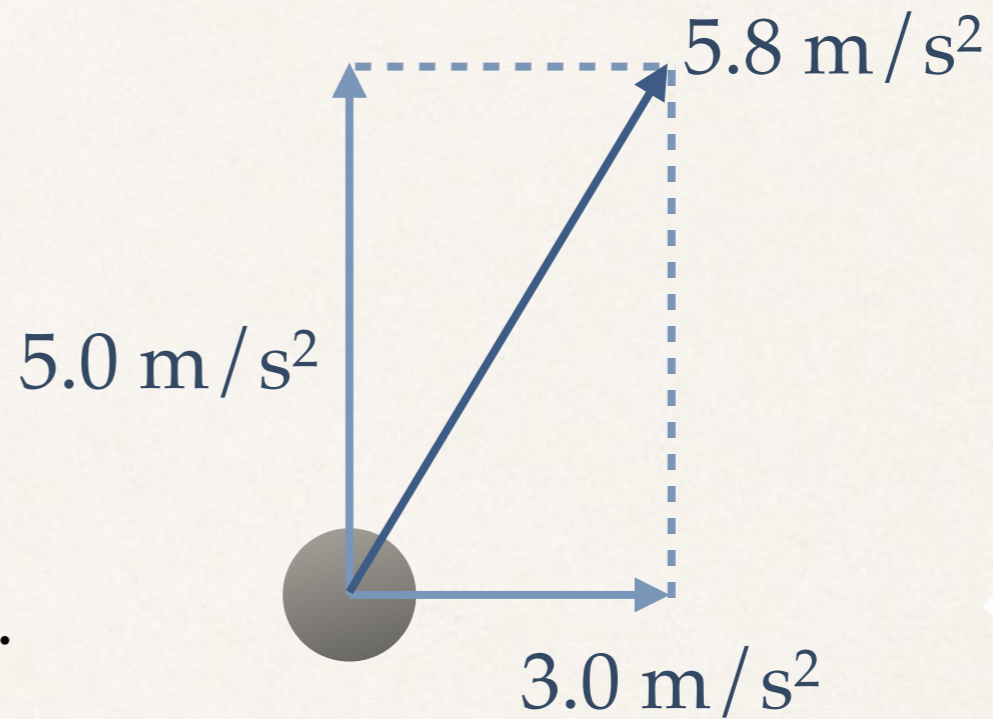
Methods of Adding Vectors



Parallelogram Method

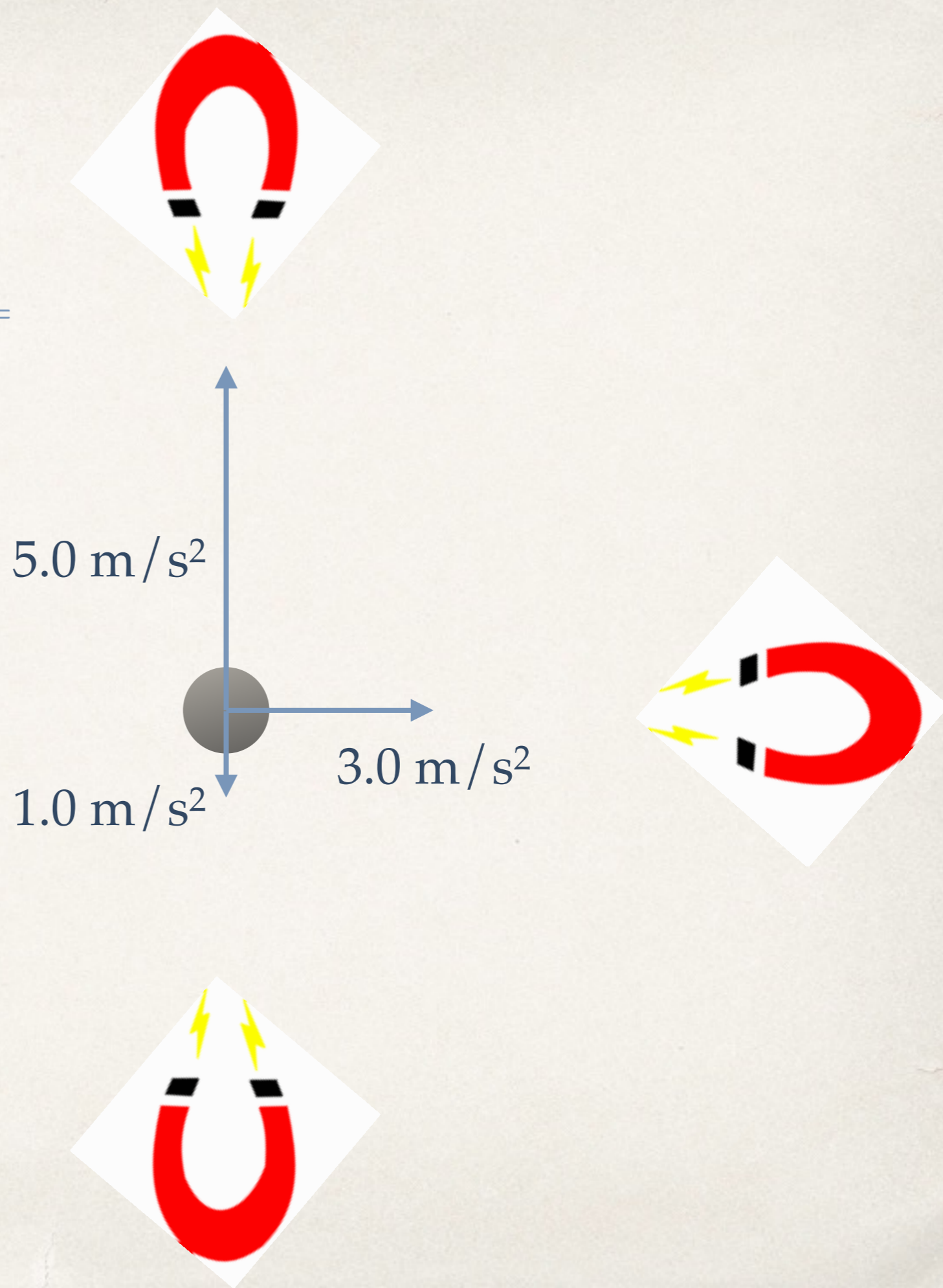
Example 1

- ❖ A metal sphere is being pulled by magnets in front of it as well as to its right.
- ❖ The magnet on right accelerates the sphere to the right at 3.0 m/s^2 . The magnet in front of the sphere accelerates it forward at 5.0 m/s^2
- ❖ What is the magnitude of the sphere's net acceleration?
- ❖ *Ans.* $a_{net} = 5.8 \text{ m/s}^2$



Example 2

- ❖ A third magnet is added behind the sphere, pulling it at a rate of 1.0 m/s^2
- ❖ What is the magnitude of the new net acceleration?



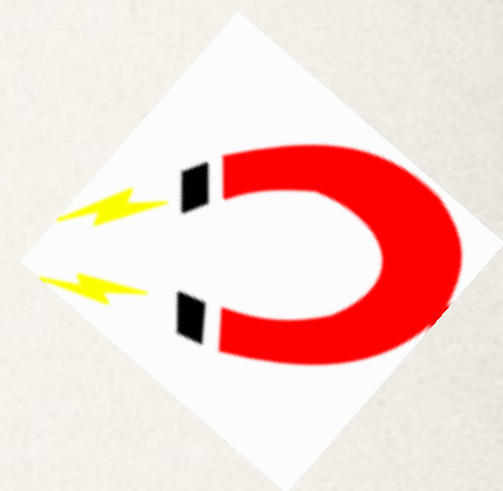
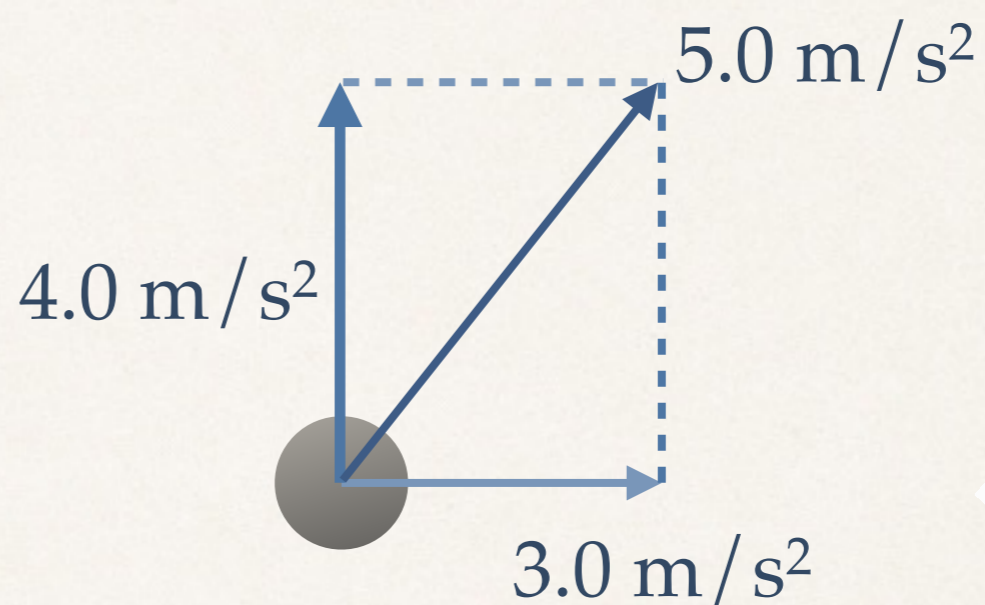
Example 2

- ❖ A third magnet is added behind the sphere, pulling it at a rate of 1.0 m/s^2
- ❖ What is the magnitude of the new net acceleration?

- ❖ $a_y = 5.0 \text{ m/s}^2 - 1.0 \text{ m/s}^2$

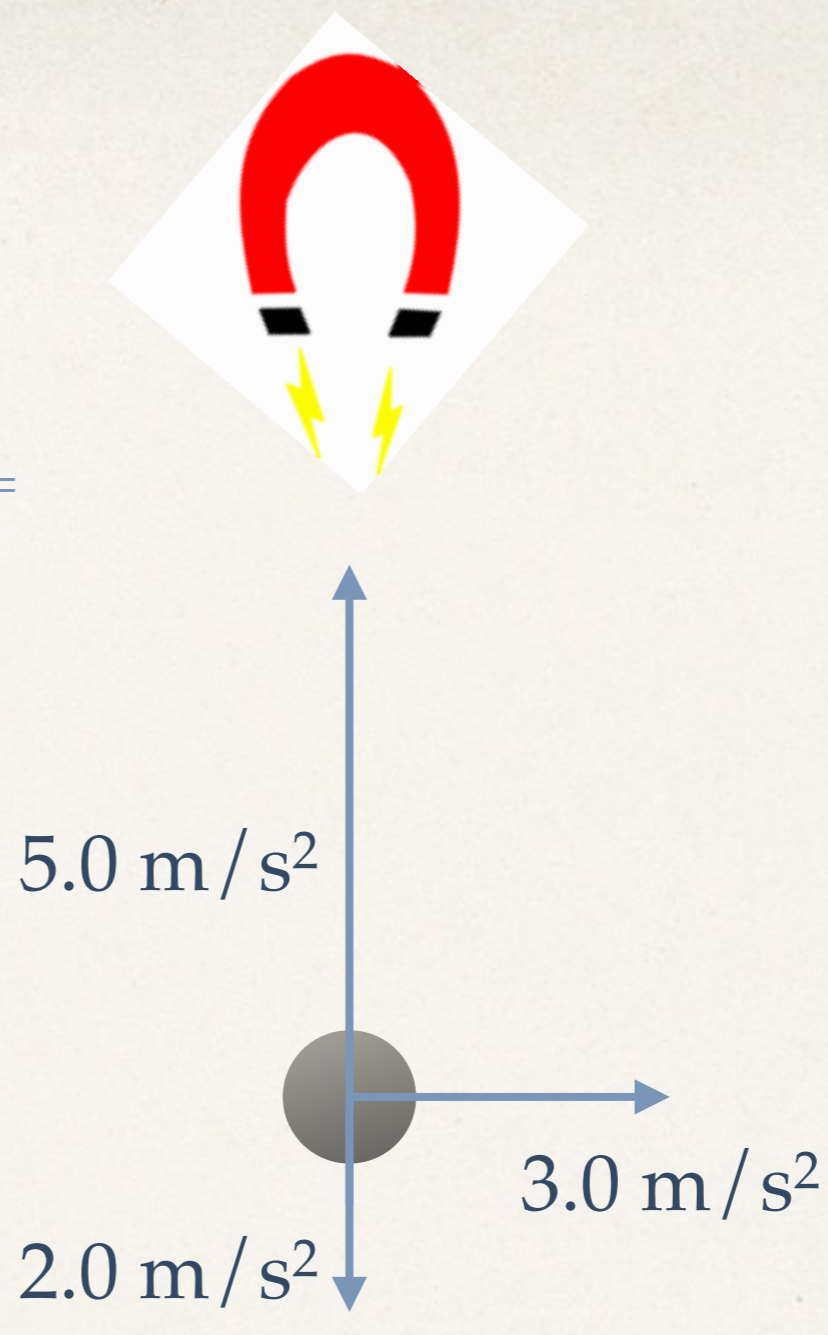
- ❖ $a_y = 4.0 \text{ m/s}^2$

- ❖ *Ans.* $a_{net} = 5.0 \text{ m/s}^2$



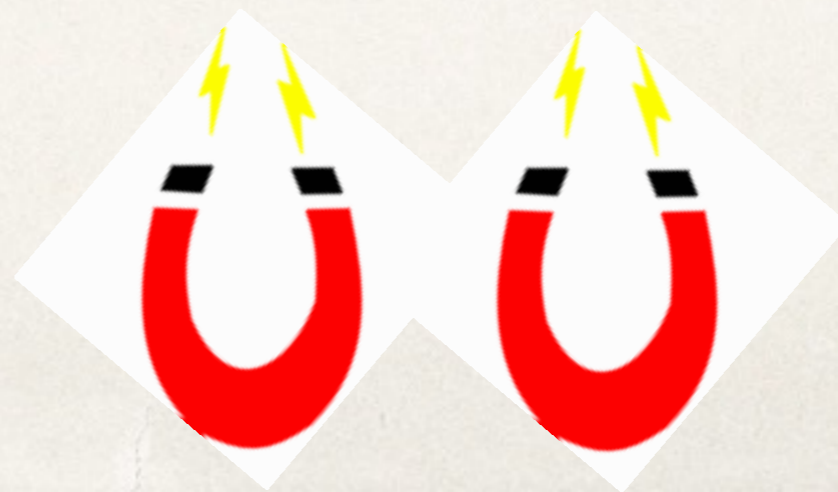
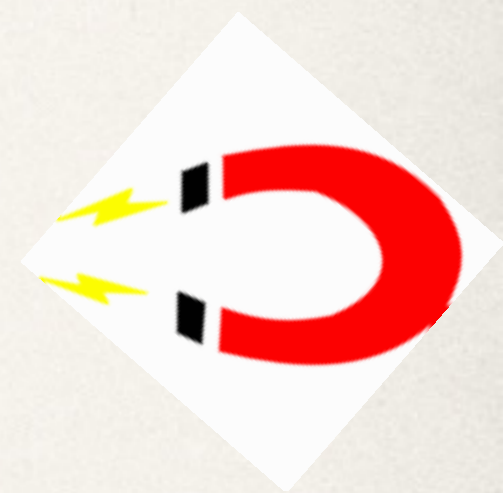
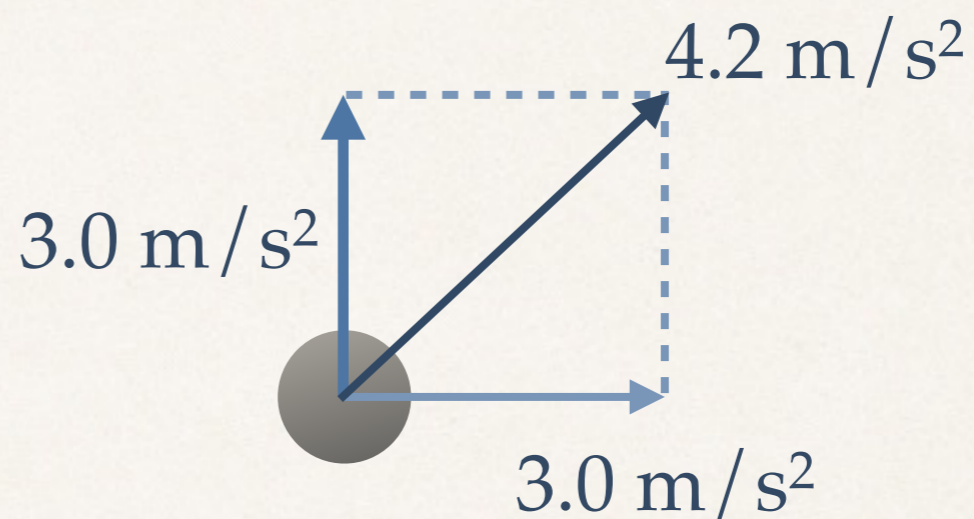
Example 3

- ❖ The magnet behind the metal sphere is doubled, doubling the rate acceleration in that direction.
- ❖ Now what is the magnitude of the net acceleration?

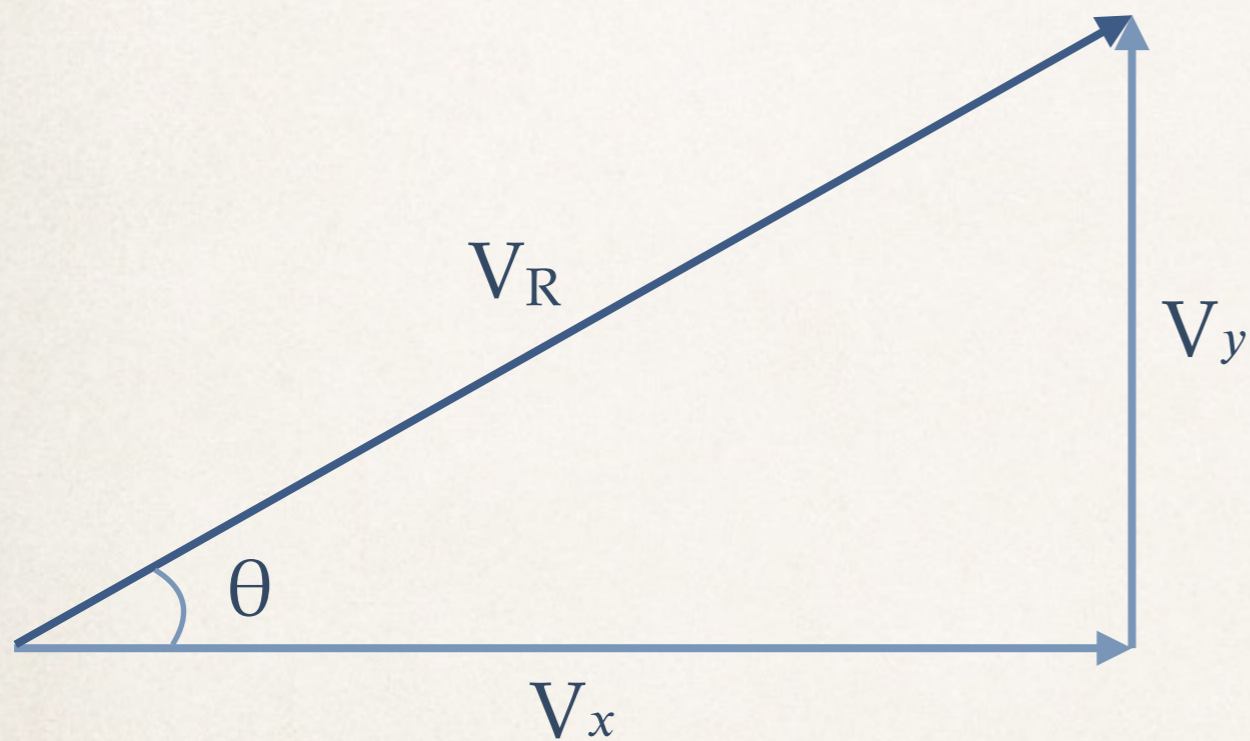


Example 3

- ❖ The magnet behind the metal sphere is doubled, doubling the rate acceleration in that direction.
- ❖ Now what is the magnitude of the net acceleration?
- ❖ *Ans. $a_{net} = 4.2 \text{ m/s}^2$*

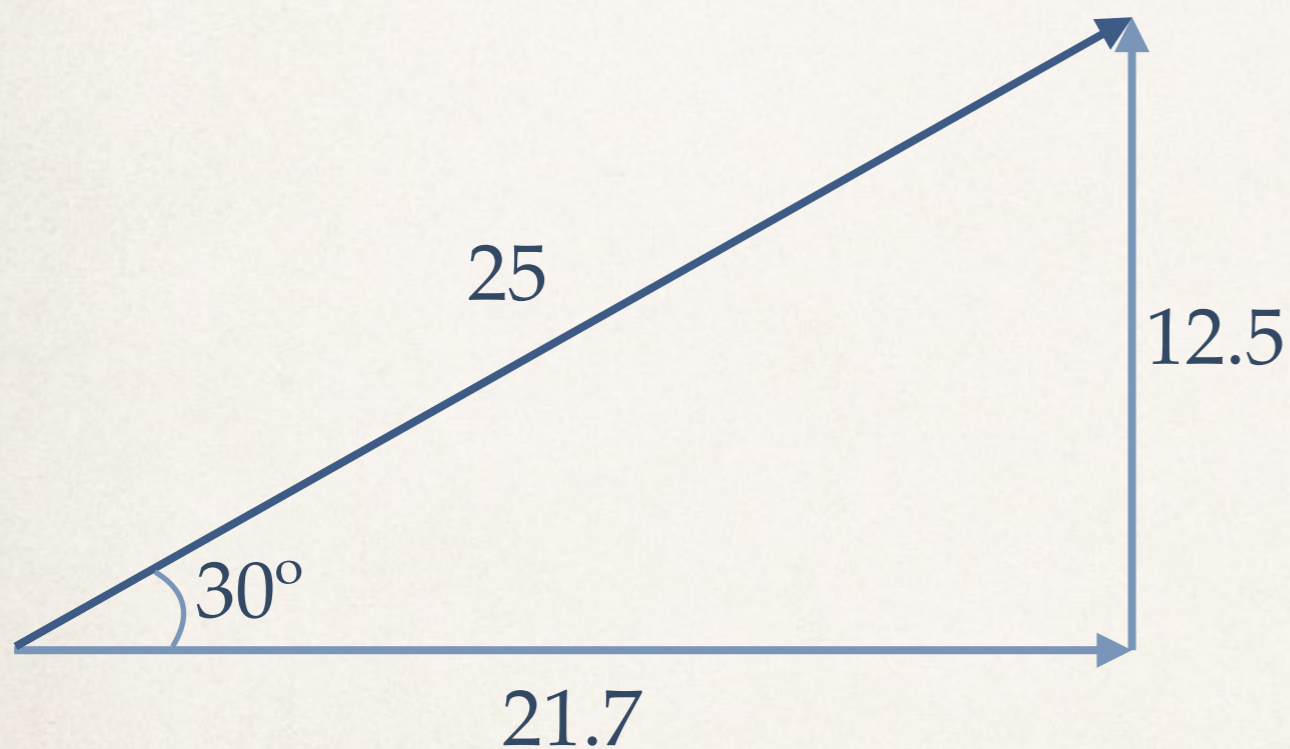


Angle of the Resultant Vector



- ❖ What is the direction of the resultant vector?
- ❖ SOH CAH TOA
- ❖ $\sin \theta = \text{opposite} / \text{hypotenuse}$
- ❖ $\sin \theta = V_y / V_R$
- ❖ $\cos \theta = V_x / V_R$
- ❖ $\tan \theta = V_y / V_x$

Angle of the Resultant Vector



- ❖ Can you break the resultant vector back into its x and y components?
- ❖ $V_x = V_R \cos\theta$
- ❖ $V_x = 25 \cos 30^\circ = 21.7$
- ❖ $V_y = 25 \sin 30^\circ = 12.5$
- ❖ **MAKE SURE YOUR CALCULATOR IS IN DEGREE MODE**

Vector Components

❖ Two ways to specify a vector in a given coordinate system:

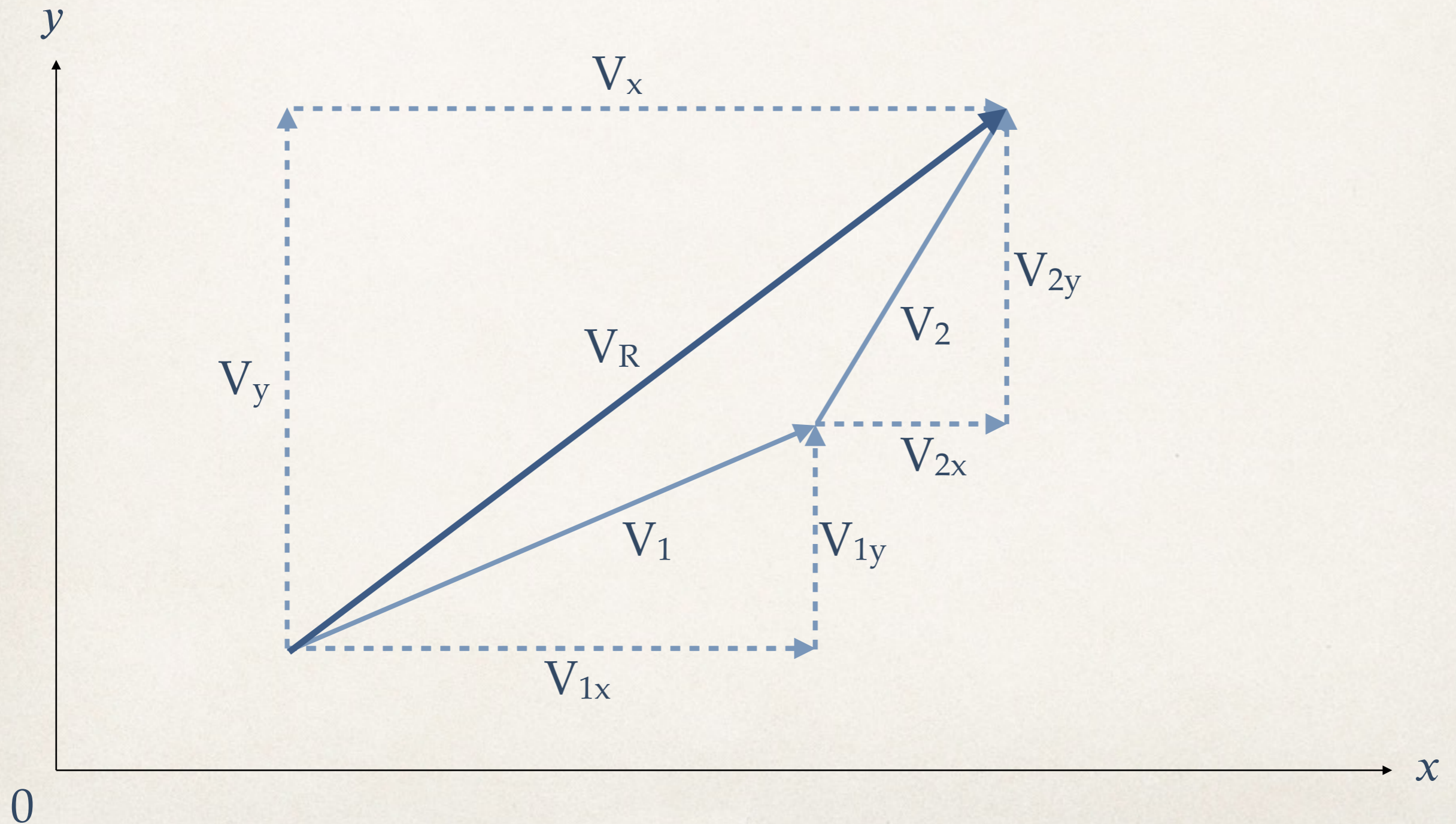
1. Give its x and y components

❖ $\mathbf{V} = 21.7\hat{x} + 12.5\hat{y}$

2. Give its magnitude V and the angle θ it makes with the positive x -axis

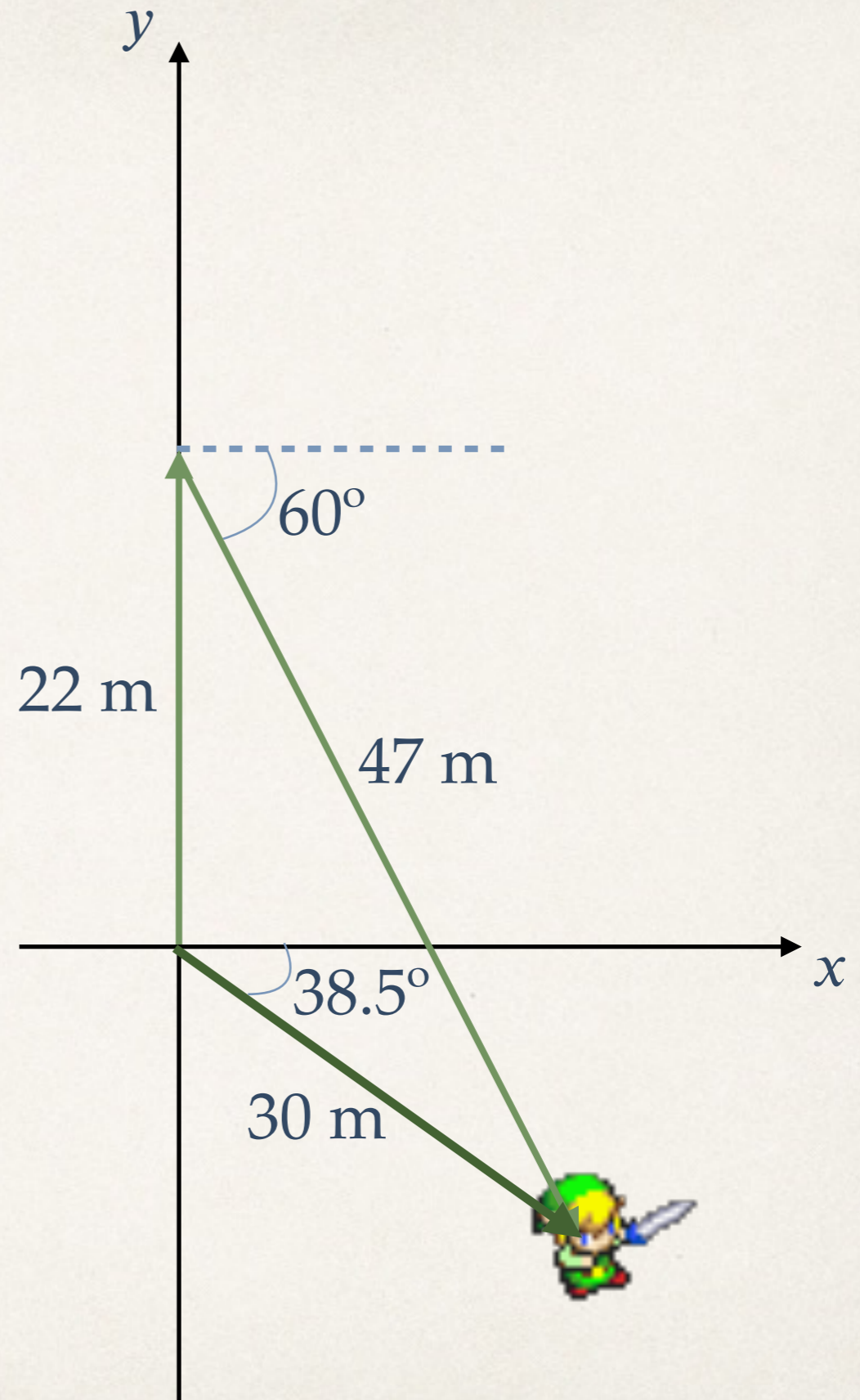
❖ $\mathbf{V} = 25 @ 30^\circ$

Vector Decomposition and Resolution



Example 4

- ❖ Link goes out to Hyrule Field to do some treasure hunting.
- ❖ Link walks 22.0 m north and plucks a hidden rupee.
- ❖ He then runs at a direction 60° south of east for 47.0 m to snag another rupee.
- ❖ What is Link's displacement from his starting point?
- ❖ *Ans. $D = 30.0 \text{ m} @ -38.5^\circ$*



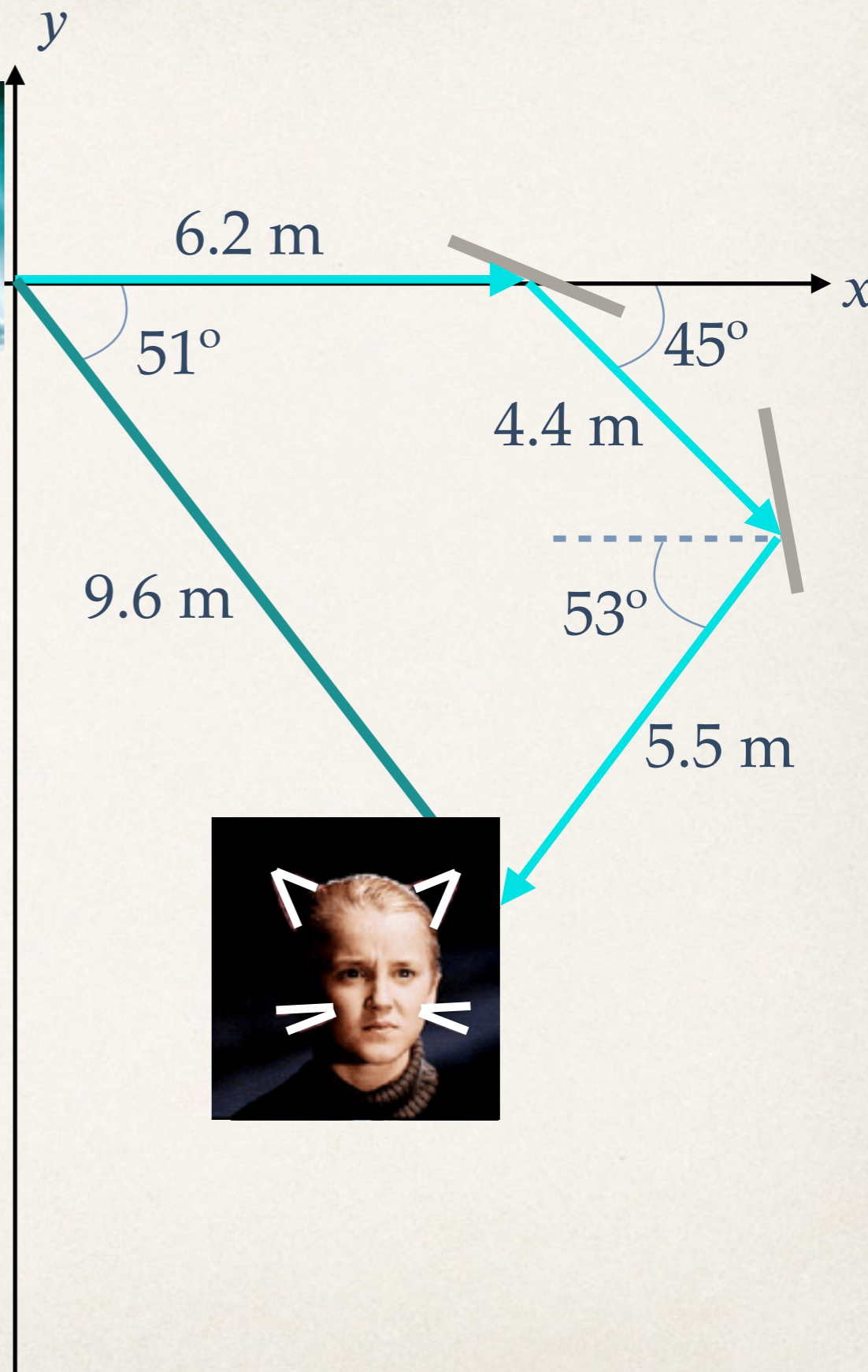
Rules for Resolving Vectors

1. Decompose all vectors into x and y components
2. Add the x components together to get the x component of the resultant.
Ditto for y
 - ❖ $\sum V_x = V_{1x} + V_{2x} + \dots$
 - ❖ $\sum V_y = V_{1y} + V_{2y} + \dots$
3. If you want to know the magnitude and direction of the resultant vector:
 - ❖ $V = \sqrt{V_x^2 + V_y^2}$ $\tan \theta = V_y / V_x$
 - ❖ *Note: pay attention to signs! Any components that point along the negative x or y axis get a negative sign*

Example 5

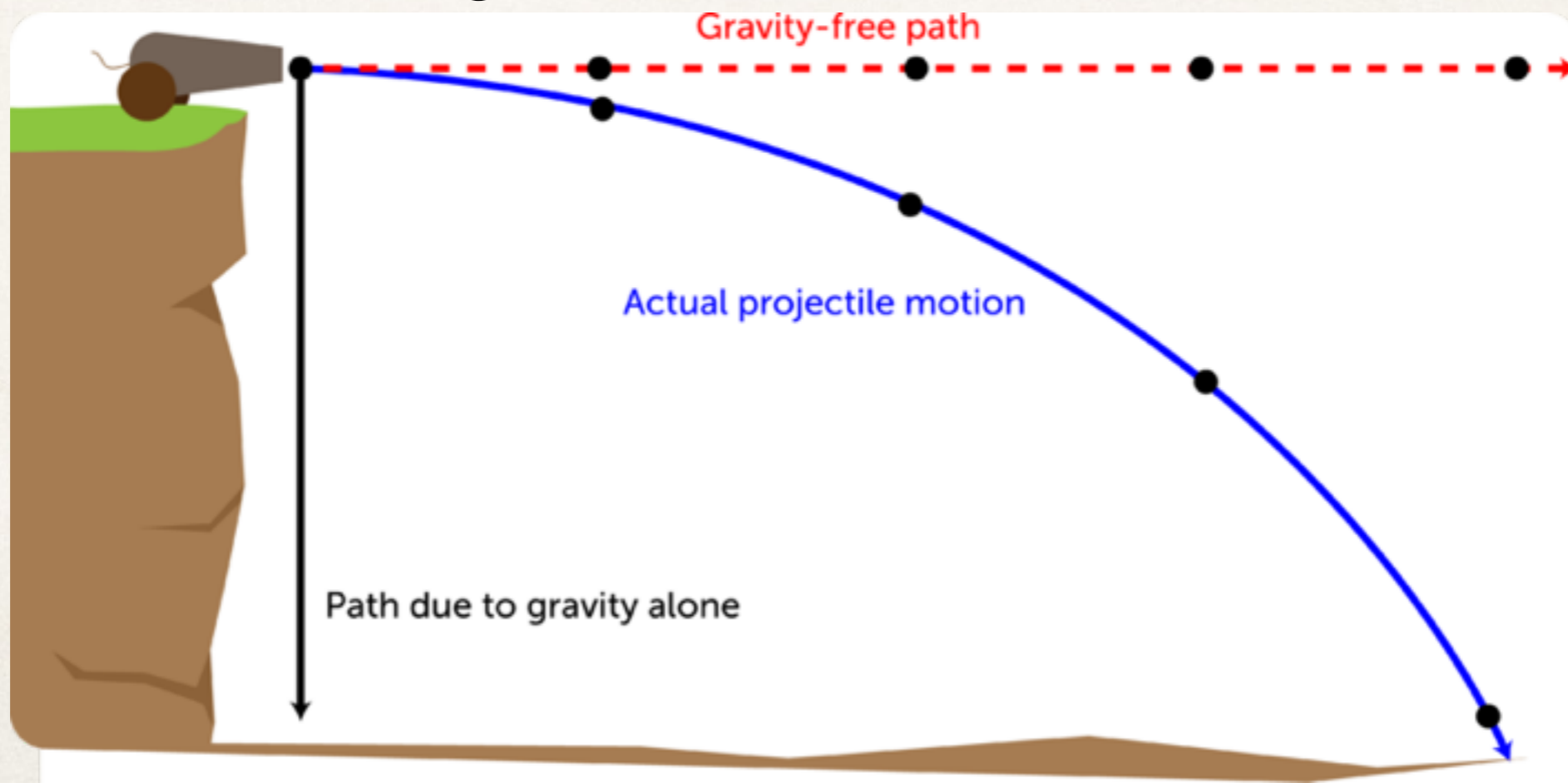


- ❖ Hermione Granger, being the clever one, uses mirrors to bounce her spell around the room and hit Draco Malfoy in his dumb face.
- ❖ Her spell rockets due east for 6.2 m, bounces southeast (45°) for 4.4 m, and reflects off a second mirror at 53° south of west for 5.5 m.
- ❖ What is the total displacement of Hermione's spell?
- ❖ *Ans. $D = 9.6 \text{ m @ } -51^\circ$*



Projectile Motion

- ❖ Much like free fall, but now the falling object also has a horizontal component to its motion as well.
- ❖ Per usual, we will ignore air resistance

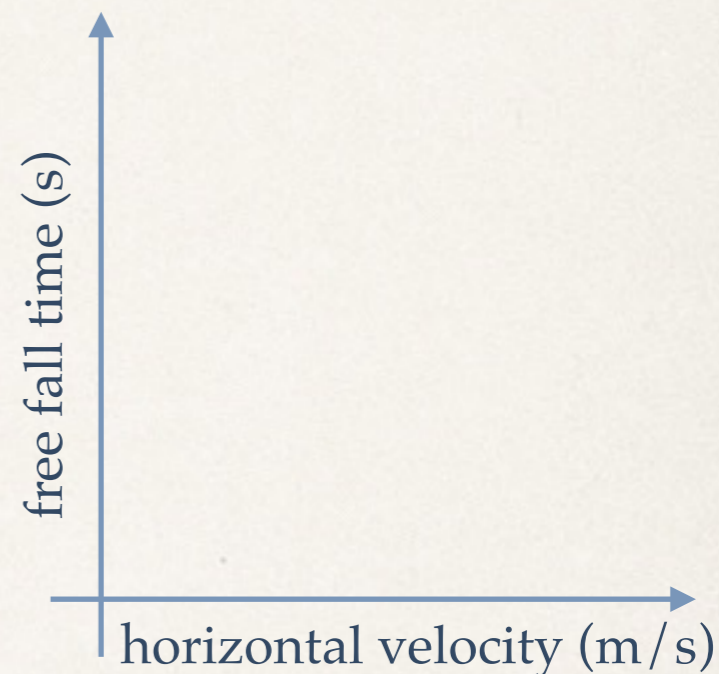


Projectile Motion

- ❖ We already know that the force of gravity imparts a vertical acceleration onto objects in free fall
- ❖ What about its horizontal velocity?

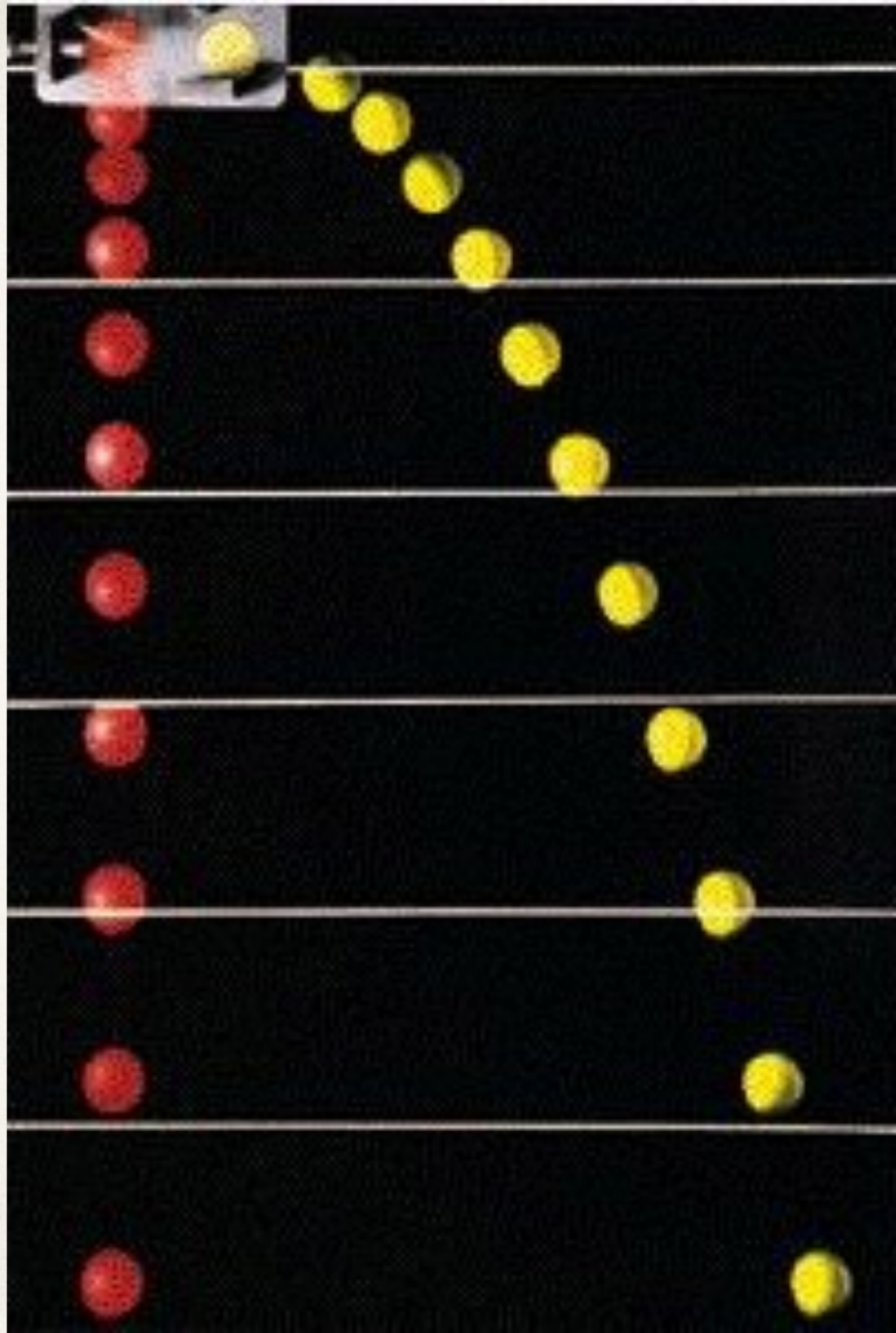
Projectile Lab 1

- ❖ Research Question: How does the horizontal velocity affect the time spent in free fall?
- ❖ Claim: (Your conclusion here)
- ❖ Evidence: Create a *time vs v_x* graph
- ❖ Justification: If there is a correlation between horizontal velocity and free time, why is it the way it is? If there isn't a correlation, why not?



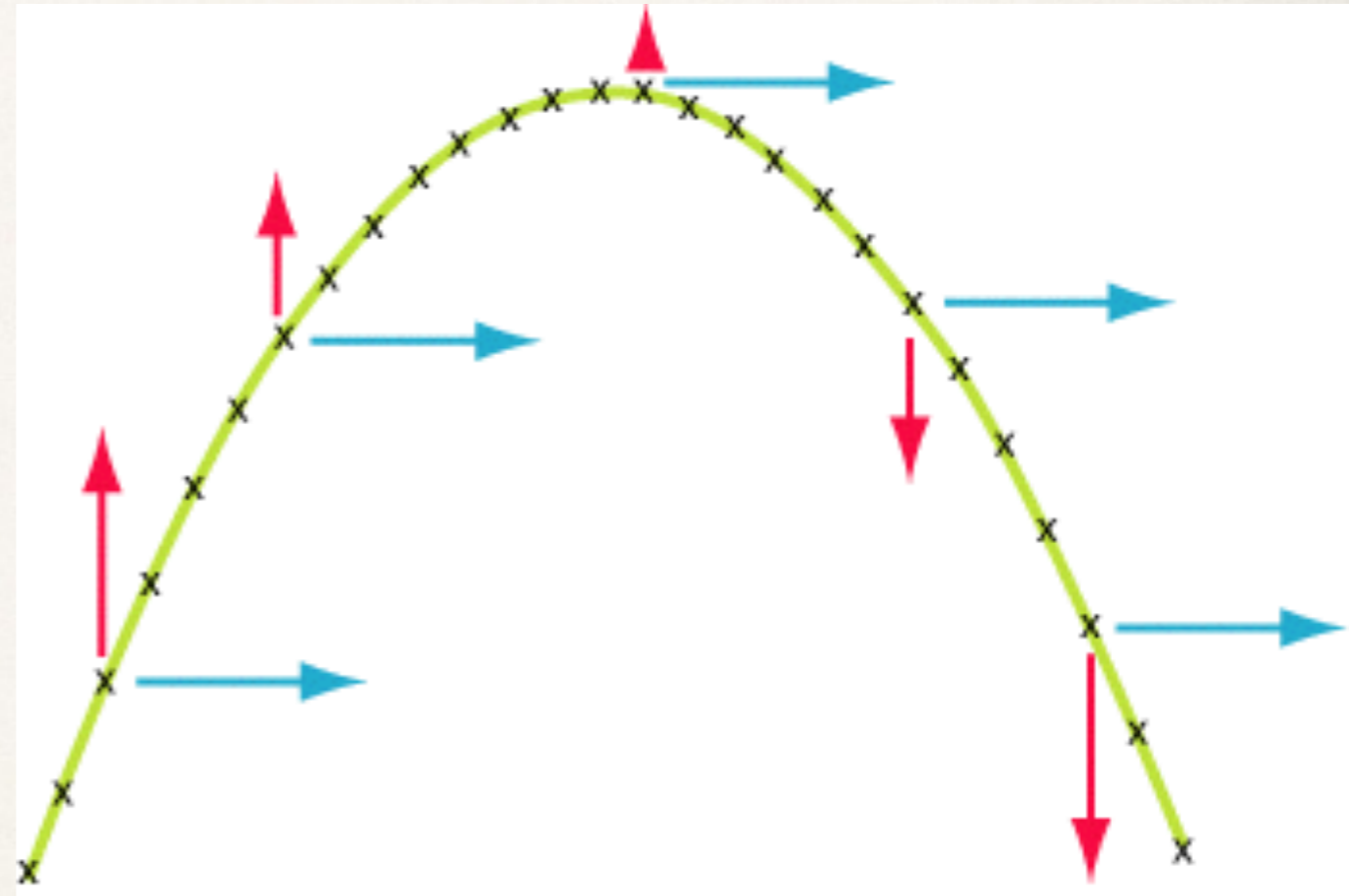
Projectile Motion

- ❖ The horizontal and vertical components of projectile motion are *completely* independent of each other
- ❖ An object projected horizontally will reach the ground at the *same time* as an object dropped vertically



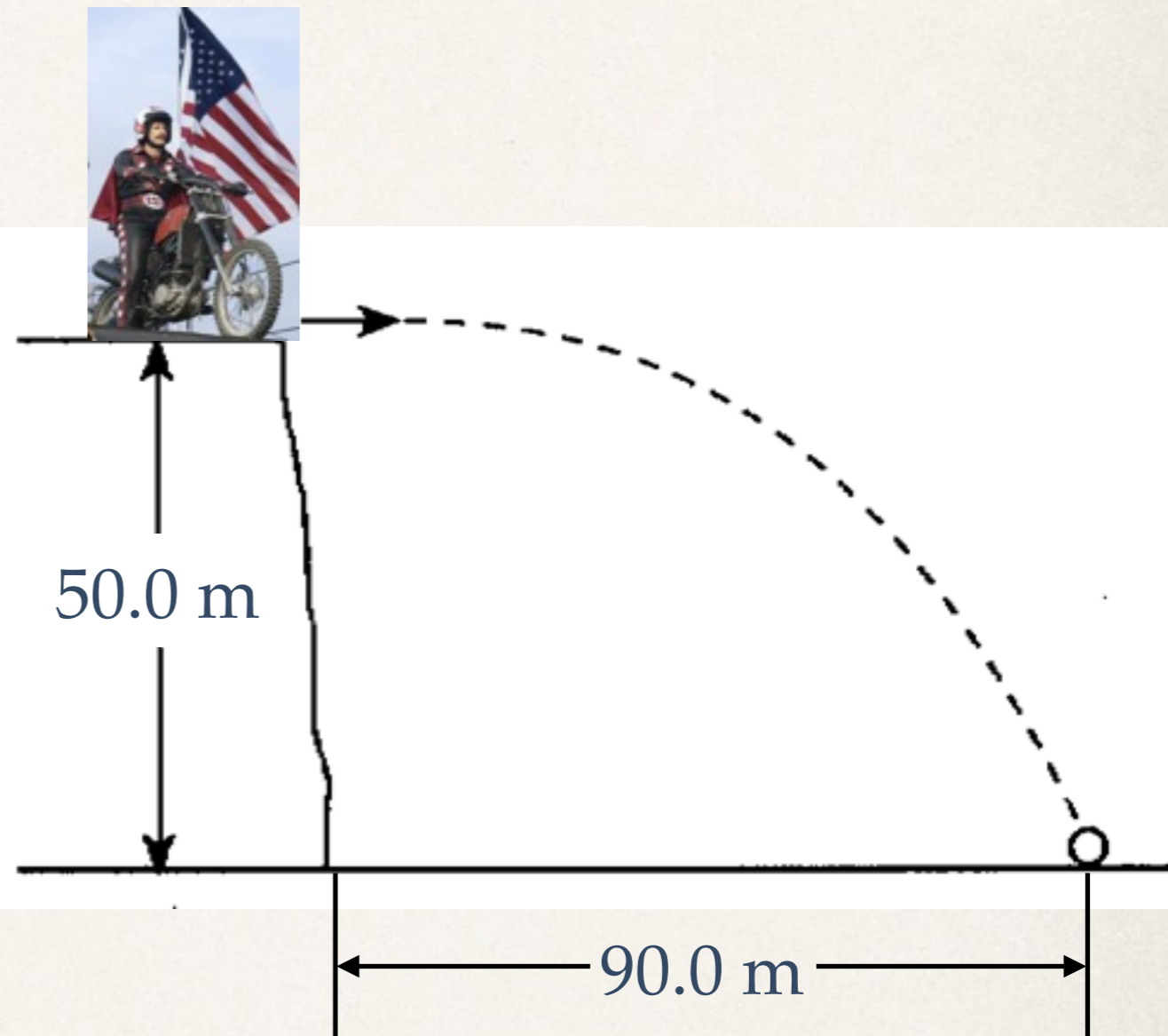
Projectile Motion

- ❖ The vertical velocity behaves *exactly* as in free fall
- ❖ The horizontal velocity will remain *constant*
- ❖ Nothing to accelerate horizontally
- ❖ No acceleration = no change in velocity



Example 6

- ❖ Rod Kimble, stuntman, drives his motorcycle off a 50.0-m-high cliff to raise money for his stepdad's heart surgery
- ❖ How fast must Rod leave the cliff-top if he is to land on level ground, 90.0 m from the base of the cliff where the cameras are?
- ❖ *Ans.* $v_{x0} = 28.2 \text{ m/s}$



Example 7

❖ Alex Wood, kickoff man for the USC Trojans, kicks the football at an angle $\theta_0 = 37.0^\circ$ with a velocity of 20.0 m/s. Calculate:

(a) the maximum height

❖ *Ans.* $y_{max} = 7.35 \text{ m}$

(b) the time of travel before the football hits the ground

❖ *Ans.* $t = 2.45 \text{ s}$

(c) how far away it hits the ground

❖ *Ans.* $\Delta x = 39.2 \text{ m}$

(d) the velocity vector at the maximum height

❖ *Ans.* $v = 16.0 \text{ m/s } x$

(e) the acceleration vector at maximum height

❖ *Ans.* $a = 9.81 \text{ m/s}^2 y$

Sanity Check

- ❖ You're riding your Vespa down the street at a constant speed when a squirrel lands in your lap! You freak out and throw the squirrel straight up in the air (from your point of view) while you and your Vespa continue to travel forward at a constant speed. If air resistance is neglected, where will the squirrel land?
 - A. behind you
 - B. in front of you
 - C. back in your lap

Sanity Check

- ❖ You're riding your Vespa down the street at a constant speed when a squirrel lands in your lap! You freak out and throw the squirrel straight up in the air (from your point of view) while you and your Vespa continue to travel forward at a constant speed. If air resistance is neglected, where will the squirrel land?
 - A. behind you
 - B. in front of you
 - C. back in your lap

Example 8

- ❖ Three cannonballs are launched with initial velocities of 30. m/s
 - ❖ One is launched at 37° above the horizontal
 - ❖ One is launched at 45°
 - ❖ The third is launched at 53°
- ❖ How far does each cannonball travel?
- ❖ *Ans. $x(37^\circ) = 88 \text{ m}$*
- ❖ *Ans. $x(45^\circ) = 92 \text{ m}$*
- ❖ *Ans. $x(53^\circ) = 88 \text{ m}$*

Example 9

- ❖ Former USC kicker, Andre Heidari drop kicks for a field goal. He kicks the ball at an angle of 37° with a velocity of 20.0 m/s (eerily similar to Alex Wood's now famous punt) but Heidari makes contact with the ball 1.00 m above the ground. How far will the ball travel before hitting the ground?

- ❖ *Ans. $\Delta x = 40.5 \text{ m}$*



Path of the Projectile

❖ We know:

$$❖ x(t) = v_{x0}t$$

$$❖ y(t) = v_{y0}t - \frac{1}{2}gt^2$$

❖ What about $y(x)$?

$$❖ y(x) = (v_{y0}/v_{x0})x - (g/2v_{x0}^2)x^2$$

❖ form of $y = ax + bx^2$

❖ Parabolic!