From "Pocket Giants: Isaac Newton" by Andrew May

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Applied Mathematics

'... to discover the forces of nature from the phenomena of motions ...'

Early in 1684, three men were having an animated discussion in one of London's fashionable coffee houses. The oldest of the trio was the most famous: Sir Christopher Wren (1632–1723), the man who had designed St Paul's Cathedral and many other fine churches in the city. Although he had made his name as an architect, Wren's earliest work had been in the field of astronomy – a subject in which he retained a keen interest. It was an astronomical problem the three men were discussing in the coffee house that day.

Also present was Robert Hooke – not as famous as Wren, but at 49 only three years younger. The third man, by contrast, was a mere youngster of 28. His name was Edmond Halley (1656–1742) and he was already making a name for himself as one of the country's leading observational astronomers. Like Hooke and Wren, Halley was a Fellow of the Royal Society.

The three men were arguing about planetary orbits. Each of them, it seems, had independently come to the conclusion that the force holding a planet in its orbit varied as the inverse square of distance. This followed from two mathematical relationships that were well established by that time – Kepler's third law and Huygens' formula for centrifugal force. Newton, of course, had accepted this line of logic as long ago as 1666, during his enforced vacation at Woolsthorpe.

The three men agreed about something else as well. They knew that the resulting orbit was an ellipse, according to Kepler's first law. But was there a cause-and-effect relationship between these two facts? Did an inverse square law necessarily imply an elliptical orbit?

Hooke maintained that it did, and that he could prove the point mathematically. The other two were sceptical: they knew from experience that Hooke was far from being a mathematical genius. To call his bluff, Wren offered him a prize worth 40 shillings if he could provide a mathematical proof within two months. To no-one's surprise, the deadline came and went without a response.

But the problem didn't go away. Young Halley, for one, couldn't stop thinking about it. What shape of orbit results from an inverse square law of force? It was a simple enough question, and he wanted an answer to it.

Halley was acquainted with the brilliant, if slightly eccentric, Professor of Mathematics at Cambridge University. Maybe he would be able to help? Halley made the journey to Trinity College in August 1684 and paid a visit to Newton. After explaining the background, he took a deep breath and asked him the big question: How would a planet move if it were subject to a force that varied as the inverse square of distance?

Without hesitation, Newton answered that the planet would follow an elliptical orbit. When Halley asked him how he could be so sure, he was astonished to hear Newton's reply: 'Why, I have calculated it.'[16] Unfortunately, even after much frantic searching, Newton couldn't lay his hands on the proof he was sure he had written down somewhere. He promised Halley that he would redo the calculations and send them on to him later.

Newton was as good as his word. In November 1684 he sent Halley a nine-page manuscript written in Latin, entitled 'De Motu Corporum in Gyrum', 'On the Motion of Bodies in Orbit'. Despite its brevity, the manuscript did more than simply answer Halley's original question. Using standard geometrical methods, Newton succeeded in deriving all three of Kepler's laws of planetary motion, starting from a few basic assumptions about the forces involved.

Halley was impressed. He wrote to Newton asking him to put a final polish on the manuscript, so that it could be published by the Royal Society. That, Halley assumed, would be the work of a few weeks at most. He was wrong. As it turned out, he had to wait more than a year before he saw the revised manuscript. It was worth the wait. By the time Newton had finished, his nine-page manuscript had grown into the greatest scientific treatise the world had ever seen.

Newton hadn't planned it that way. He was still irritated by the whole subject of 'natural philosophy', viewing it as something he had found interesting in the past but from which he had long since moved on. He would have been happy to get Halley's manuscript out of the way as quickly as possible. But as he started to work on it, he saw vistas of previously undreamed-of possibility opening up before him. The same line of thinking he had used to connect mathematics to planetary motion could be applied to other things as well – possibly even to *everything*. The manuscript started to grow – and so did Newton's enthusiasm.

He began to perceive that 'the whole difficulty of philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces'.[17] Sir Francis Bacon had said much the same thing sixty years earlier but Newton made a crucial extra step. He recognised that this entire cycle of discovery and demonstration could be achieved through the medium of mathematics. It was the great breakthrough the scientific world had been waiting for.

It took Newton eighteen months to write the book he called *Philosophiae Naturalis Principia Mathematica* – 'Mathematical Principles of Natural Philosophy'. The very title points to an astonishing revolution in world view. In the seventeenth century, mathematics was seen as elegant, ordered and predictable. Natural philosophy – the study of the physical world – was none of those things. Or at least not until Isaac Newton got hold of it.

It is virtually certain that Newton derived all the main results of the *Principia* using his 'secret' method of fluxions – calculus, in other words. But he wanted to keep that technique to himself, so he laboriously translated all the proofs into traditional geometric form for publication. He gave the *Principia* the standard format of an ancient mathematical treatise, complete with theorems, propositions, proofs and corollaries. Coupled with the fact that it was written in Latin, the book's turgid style would have looked old-fashioned and abstruse even in Newton's time.

This was no accident. If the *Principia* was difficult to read, it was because that is how the author planned it. Newton was still pathologically averse to criticism of any kind. He was convinced the only reason anyone could have for faulting his work was that they were too ignorant to understand it. So what could he do to minimise the number of ignorant people who would read the book? Making it as obscure as possible would certainly help him 'to avoid being baited by little smatterers in mathematics',[18] as he explained later.

The *Principia* is organised in three sections which Newton called 'books' even though it was published as a single volume. The first book is a greatly expanded version of the earlier manuscript he sent to Halley, 'De Motu Corporum in Gyrum'. Book 2 deals with the motion of bodies when they are immersed in a surrounding medium rather than a vacuum – including, *inter alia*, a detailed mathematical refutation of Descartes' theory of vortices. It is the third book which finally reveals the true scope and power of the Newtonian approach. Called 'De Mundi Systemate', 'On the System of the World', it describes the practical application of the theory developed in the first two books to the motion of the Moon, the planets, the satellites of Jupiter, several recently observed comets and even the previously baffling phenomenon of the Earth's tides.

The genius of the *Principia* lies in the way Newton applies rigorous mathematical methods to the hitherto intractable problems of the physical world. He follows a logical train of thought all the way from first principles through to observable phenomena. Even here, he was standing on the shoulders of giants. For the most part, the 'first principles' he drew on had already been propounded by others. Concepts such as inertia and momentum would have been familiar to Galileo half a century earlier.

At the outset of Book 1, Newton lays out his three laws of motion:

> Law 1: Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.
> Law 2: The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
> Law 3: To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Perhaps the most significant feature of these three laws is that each of them flies in the face of Aristotelian 'common sense'.

Aristotle stated that moving objects tend to slow down unless you apply a force to keep them going – which seems to accord with our everyday experience. But here was Newton claiming the very opposite: that an object will continue to move *unless* you apply a force to slow it down.

Even today, many science-fiction movies can be counted on to break all three of Newton's laws at some point during the special effects sequences. There is a good reason for this. If the special effects people strictly obeyed Newton's laws, most members of the audience would think they had made a mistake. That's how counter-intuitive the first three statements in the *Principia* are!

The other big idea Newton introduced in the *Principia* was his famous 'Law of Gravity' (see Glossary): every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. Using calculus – carefully disguised as classical geometry – he demonstrated that a spherical body such as the Sun or a planet will exert exactly the same gravitational attraction as a single point, of the same mass, located at its centre.

What made Newton's work different from anything that had gone before was its predictive power. His talk about forces and orbits was precision mathematics. He did not have to limit himself to approximating planetary orbits as circles, or planetary bodies as perfect spheres. He could treat elliptical orbits and non-spherical bodies with exactly the same mathematical apparatus.

And he could go further still. For the first time in history, he found a convincing way to explain the tiny perturbations astronomers had observed in the motion of the Moon. An ellipse is the orbit you get if you focus solely on the Moon going round the Earth, in isolation from the rest of the universe. In reality, the situation is more complicated than that. At the very least, you have to consider the three-body system of Earth, Moon and Sun. When Newton did so, he found that the Moon's orbit turned into a perturbed ellipse very much like the one that is actually observed.

Book 1 of the *Principia* was finished by April 1686, and Newton duly sent the manuscript off to the Royal Society. Its president at the time was none other than Samuel Pepys (1633–1703), a man best known today for his posthumously published diary. In his lifetime, however, Pepys had a reputation as a competent administrator, and it was on this basis – rather than any great scientific achievement – that he had secured the presidency of the Royal Society. In this capacity, Pepys agreed to provide the imprimatur for the publication of the *Principia* on one condition: someone else was going to have to pay for it. The Society was short of cash and was not prepared to risk as much as a penny towards the cost of publishing so abstruse a book.

That might have been the end of the *Principia*, if the man who proposed it in the first place, Edmond Halley, had not come to the rescue. He offered to bear all the costs of publication himself and even undertook to edit the entire text single-handed.

As far as Newton was concerned, things were looking up. Little did he suspect that a long-forgotten argument was about to resurface.

It was only to be expected that Hooke, as an officer of the Royal Society, would find out sooner or later that his long-time rival was about to publish a new book about planetary motions. Hooke automatically assumed the book had grown out of his own earlier discussions with Newton. That was not an unreasonable assumption; there may even have been some truth to it.

It was equally reasonable, under the circumstances, that Hooke should feel Newton owed him an acknowledgement for his role in the process. But what was his role, exactly? Hooke had one view on the subject; Newton had another. Hooke expressed his view in a letter to Halley, and the latter duly passed it on to Newton: 'Mr Hooke has some pretensions upon the invention of the rule of the decrease of gravity. He says you had the notion from him and seems to expect you should make some mention of him in the preface.'[19]

Newton was appalled by the suggestion. He felt that Hooke's contribution had been trivial and that he had done all the hard work himself. He complained to Halley that if Hooke had his way, 'Mathematicians that find out, settle and do all the business must content themselves with being nothing but dry calculators and drudges and another that does nothing but pretend and grasp at all things must carry away all the invention.'[20]

This reaction is interesting. It gets close to acknowledging that Hooke did indeed come up with a few ideas of his own, but that he just threw them out in scattershot fashion without knowing how to develop or test them. As far as Newton was concerned, if he picked up one of Hooke's discarded ideas and made it work, then the credit should be due to him. It is a contentious point. Arguments of this type often crop up in creative endeavours involving more than one individual. Some people will agree wholeheartedly with Newton; others may feel that Hooke had a point.

One thing was beyond dispute, however, Newton was never going to submit to what he regarded as intellectual blackmail. He was so annoyed he threatened to stop work then and there, before he had completed the third book. He wrote to Halley: 'Philosophy is such an impertinently litigious lady, that a man had as good be engaged in lawsuits, as have to do with her. I found it so formerly, and now I am no sooner come near her again, but she gives me warning.'[21]

Fortunately for posterity, Halley eventually managed to calm Newton down. The *Principia* finally saw print in the summer of 1687. Tucked away inside was a brief statement to the effect that the inverse square law had previously been 'severally observed' by Hooke, Wren and Halley.