Answer Key

8. A function f is defined by the parametric equations x = 4cos(t) and y = 4sin(t). Find the area enclosed by the graph of f.

   x^2 = 16cos^2(t)
   y^2 = 16sin^2(t)
   x^2 + y^2 = 16
   Graph is a circle with radius 4
   A = π*radius^2
   A = 16π

9. A subatomic particle's velocity at time t is defined by the vector << 4/(t^3), e^-t >>. Find the distance traveled by the particle from time t = 4.20 to time t = 42.0.

   X-velocity = dX/dt = 4/(t^3)
   Y-velocity = dY/dt = e^-t

   $$\text{Distance} = \int_{4.2}^{42} \sqrt{(4/t^3)^2 + (e^{-t})^2}\, dt$$

   = 0.1137

10. Write, but do not evaluate, an expression describing the rate of lengthening of a cube's diagonal as a function of rate of growth of the cube's volume.

    X = Diagonal = side * $\sqrt{3}$
    side = diagonal/ $\sqrt{3}$
    Volume = side^3 = (diagonal/ $\sqrt{3}$)^3 = diagonal^3/(3 $\sqrt{3}$)
    dV/dt = (3*X^2/(3 $\sqrt{3}$))dX/dt

$dX/dt = \sqrt{3}/x^2 * dV/dt$

11. Let R be the region in the first quadrant bounded by the graph of y = (x+2)^2 + 4 and y = 10. Find the volume of the solid generated when R is rotated around the x-axis.

10 = (x+2)^2 + 4

X+2 = + $\sqrt{6}$ or - $\sqrt{6}$

X = positive => $\sqrt{6}$ - 2

A = (10^2 - ((x+2)^2 + 4)^2)*π.

$$V = \int_{0}^{\sqrt{6}-2} (10^2 - ((x+2)^2 + 4)^2) * \pi \, dx$$

= 27.2134

12. Given functions f(x) = 5 and g(x) = 2 + x^2, find the volume of a 3D object formed by semicircles perpendicular to the area enclosed by f(x) and g(x). The bases of the semicircles are perpendicular to the x-axis.

Area of each semicircle = $1/2 * \pi r^2$

A = ½ * π (5-(2+x^2))^2

A = ½ * π (x^2+3)^2

To find the limits of integration, find where the two equations intersect:

5 = 2 + x^2

x = $\sqrt{3}$ *and* $-\sqrt{3}$

$$V = \int_{-\sqrt{3}}^{\sqrt{3}} A \, dx$$

$$V = \int_{-\sqrt{3}}^{\sqrt{3}} ½ * \pi (x^2+3)^2$$

$$V = \tfrac{1}{2}\pi \int_{-\sqrt{3}}^{\sqrt{3}} (x\hat{}2+3)\hat{}$$

V = **58.197**

13.Find the equation of the line tangent to $r = 4\cos\theta + 4\sin\theta$ at $\theta = 2\pi/3$.

$X = r\cos\theta = 4cos^2\theta + 2sin2\theta$

$Y = r\sin\theta = 2sin2\theta + 4sin^2\theta$

$dX/d\theta = 8\cos\theta(-\sin\theta) + 4\cos2\theta = -4\sin2\theta + 4\cos2\theta$

$dY/d\theta = 8\sin\theta(\cos\theta) + 4\cos2\theta = 4\sin2\theta + 4\cos2\theta$

$dY/dX = (4\sin2\theta+4\cos2\theta)/(-4\sin2\theta+4\cos2\theta) = (-\sqrt{3}/2 + (-\tfrac{1}{2}))/(-\tfrac{1}{2}+\sqrt{3}/2)$

= -3.732

$X = 4(-\tfrac{1}{2})\hat{}2 + (-\sqrt{3}) = -0.732$

$Y = 4(-\sqrt{3}/2)\hat{}2 + (-\sqrt{3}) = 1.268$

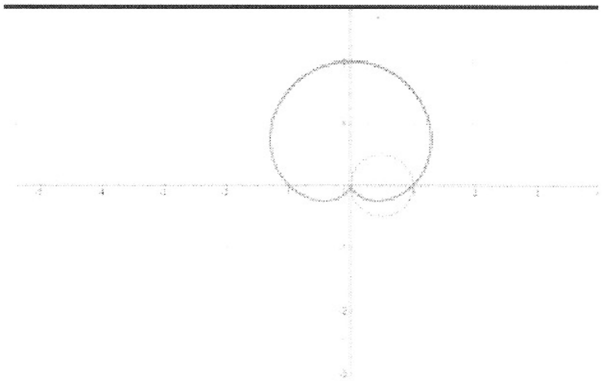
1.268 = -3.732*-0.732 + b

b=-1.464

Y = -3.732X -1.464

14.Find the area enclosed by $r = \cos\theta$ and $r = \sin\theta - 4$.

Graph:



The easiest way to solve this (in my opinion) is to find the entire area of the circle then subtract the area between the limacon and the circle.

Area of circle = πr^2

$A_c = \pi\,(.5)^2 = .785$

To find the area between the limacon and circle:

$$A_2 = \tfrac{1}{2} \int_{\pi/2}^{\pi} (\cos\theta - \sin\theta + 1)^2 \, d\theta$$

A = ½ * 0.698

A = 0.349

A_{final} = .785 - .349 = **.436**