1.a length of curve : $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$

$\frac{dx}{dt} = \frac{dIn(t)}{dt} = \frac{1}{t}$ $\frac{dy}{dt} = \frac{dsin(x)}{dt}$ =cost

$\int_1^{\pi} \sqrt{\frac{1}{t^2} + cos^2 t}\ dt = 1.7737$

b. Area under the curve : $\int_a^b y\ dx$

t= e^x y=sin (e^x) $[In(1), In(\pi)]$

$\int_0^{In(\pi)}$ sin (e^x) dx=0.9059

C. slope : $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{t}}{cost}$ =-0.3183 when t= π

2. a. $a_y = \frac{d(v_y)}{dt} = \frac{d\left(\frac{dy}{dt}\right)}{dt} = \frac{dt^2}{dt}$ =2t t=1s a_y =2m/ s^2

b. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{t^2}{\frac{1}{(t^2+1)}} = t^2\left(t^2+1\right)$ t=0.75s $\frac{dy}{dx} = 0.88$

y=0.88x

C. $a_x = \frac{d(v_x)}{dt} = \frac{d\left(\frac{dx}{dt}\right)}{dt} = \frac{d\left(\frac{1}{1+t^2}\right)}{dt}$ =-2t(1+t^2)^(-2)

$a_y = 2t$

x= $\int \frac{1}{1+t^2}$ dt=tan(t)

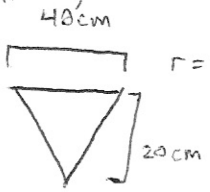
y= $\int t^2 dt$ =1/3t^3

3. ~~a. $\frac{dr}{dt}$ =0.00005m/s~~

3a. $\frac{dr}{dt} = -0.00005\ m/s$

$\frac{7}{8} \cdot V_0 = \frac{4}{3}\pi(3m)^3 \cdot \frac{7}{8} = \frac{63\pi}{2}$, r when $V = \frac{63\pi}{2}$ is $\sqrt[3]{\frac{189}{8}} = 2.869\ m$

$\frac{\Delta r}{\left(\frac{dr}{dt}\right)} = \Delta t =$ ~~(3m − 2.869m)/−0~~ $= \frac{2.869m - 3m}{-0.00005 m/s} = \boxed{2612s}$

3b. 

r = 20cm initially

$\frac{dSA}{dt} = d(\pi r^2) = 2\pi r \cdot \frac{dr}{dt} = -0.01\ cm/s$ $\quad SA_0 = \pi(20cm)^2 = 400\pi\ cm^2$

$\frac{1}{4} h_0 = 5\ cm$, @ 5cm, the surface area is $25\pi\ cm^2$ as the proportion $\frac{r}{h}$ is constant.

$\frac{\Delta SA}{\frac{dSA}{dt}} = \Delta t = \frac{25\pi\ cm^2 - 400\pi\ cm^2}{-0.01\ cm/s} = \boxed{37,500s}$

$v=\frac{4}{3}\pi(R-r)^3$

$\frac{dv}{dt}=4\pi(R-r)^2\frac{dr}{dt}$

Since $v=\frac{4}{3}\pi R^3=\frac{4}{3}\cdot\pi\cdot 3^3=113.04$,

$\frac{1}{8}v=\frac{4}{3}\pi\cdot\left(\frac{R}{2}\right)^3$ $r=\frac{R}{2}$

$\frac{113.04}{dt}=4\pi\cdot\left(\frac{3m}{2}\right)^2\times 0.00005m/s$

$dt=80000$

attached on back

4. a. $\int_{-2}^{2} x^2\,dx=\frac{16}{3}$

b. $y=x^2$ $x=\sqrt{y}$ from 0 to 4

$\int_{0}^{4}\pi(\sqrt{y})^2\,dy=67.02$

c. $\sin(x)+3=x^2$

Intersection are x=-1.418 , 1.979

$\int_{-1.418}^{1.979}\left|x^2-(\sin(x)+3)\right|dx=7.206$

d. $\int_{-1.418}^{1.979}\pi\cdot(\sin(x)+3)^2-\pi\cdot(x^2)^2\,dx=89.3967$

5. a. The intersections of $y=x^2-1$ *and*

$y=-\frac{1}{2}x^2+1$ are x=-1 and x=1

$\int_{-1}^{1}\left(\frac{1}{2}x^2+1\right)-(x^2-1)\,dx=3$

b. $\int_{-1}^{1}\frac{1}{2}\cdot\pi r^2\,dx=\int_{-1}^{1}\frac{1}{2}\cdot\pi\left(\frac{(-\frac{1}{2}x^2+1)-(x^2-1)}{2}\right)^2dx=5.0658$

c. $r=\sqrt{2y+1}$

$R=\sqrt{y+1}$

$\int_{0}^{1}\frac{1}{2}\cdot\pi\cdot(2y+1)\,dy+\int_{-1}^{0}\frac{1}{2}\cdot\pi\cdot(y+1)\,dy=3.9269$

6. a. $r(\theta)=3-4\sin\theta$

$y=r\cdot\sin\theta=3\sin\theta-4\sin^2\theta$

$x = r \cdot \cos\theta = 3\cos\theta - 4\sin\theta \cdot \cos\theta$

b. $\frac{dy}{d\theta} = 3\cos\theta - 8\sin\theta \cdot \cos\theta$

$\frac{dx}{d\theta} = -3\sin\theta + 4\cos^2\theta - 4\sin^2\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3\cos\theta - 8\sin\theta\cdot\cos\theta}{-3\sin\theta + 4\cos^2\theta - 4\sin^2\theta}$

$\theta = \frac{\pi}{4} \quad \sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$

$\frac{dy}{dx} = 0.89$

C. $\theta = \frac{\pi}{4} \quad r = 3 - 2\sqrt{2}$

$y = \frac{3}{2}\sqrt{2} - 2 \quad x = \frac{3}{2}\sqrt{2} - 2$

$\left(\frac{3}{2}\sqrt{2} - 2, \frac{3}{2}\sqrt{2} - 2\right)$

$y = 0.89x + b$

$b = \left(\frac{3}{2}\sqrt{2} - 2\right) - \left(\frac{3}{2}\sqrt{2} - 2\right) \cdot 0.89 = 0.23$

7. a.

$r = \pm\sqrt{4\sin(2\theta)} = 0, \ \theta = 0, \frac{\pi}{2}$

$A = \int_a^b \frac{1}{2} \cdot r^2 d\theta$

$= 4 \times \int_0^{\frac{\pi}{2}} 4\sin(2\theta)\, d\theta = 8$

b.

$3\sin(2\theta) = 0, \ \theta = 0, \frac{\pi}{2}$

$A = \int_a^b \frac{1}{2} \cdot r^2 d\theta$

$= \int_0^{\frac{\pi}{2}} \frac{1}{2} \cdot (3\sin(2\theta))^2 d\theta$

$= 3.534$

c. graph the two functions to understand the solution to this problem. Remember to zoom in, they do not perfectly overlap. Some of $r^2 = 4\sin(2\Theta)$ lies outside of $r = 3\sin(2\theta)$ in the 1st and 3rd quadrants.

$\frac{\pi}{2}$

$3sin(2\theta) = \pm\sqrt{4sin(2\theta)}$, θ = 0,0.230276996 rad, 1.361684 rad

$$A_1 = \int_{1.361684}^{\frac{\pi}{2}} \frac{1}{2} \cdot 4sin(2\theta)\,d\theta - \frac{1}{2} \cdot (3sin(2\theta))^2 = 0.332$$

$$A_2 = \int_{0}^{\frac{\pi}{2}} \frac{1}{2} \cdot (3sin(2\theta))^2 - \frac{1}{2} \cdot 4sin(2\theta)\,d\theta$$

=3.534-2 =1.534

$A_2 + 2A_1$ = Total area = 2.198