

Jenny Wang
Katrine Madsen
Period 3

Review Test

1. Eliminate the Parameter.

$$x = 5 \cos(t)$$

$$y = \frac{4}{3} \sin(t)$$

$$X = 5 \cos(t)$$

$$X^2 = 25 \cos^2(t)$$

$$Y = \frac{4}{3} \sin(t)$$

$$Y^2 = \frac{16}{9} \sin^2(t)$$

$$\frac{X^2}{25} = \cos^2(t)$$

$$Y^2 = \frac{16}{9} (1 - \cos^2(t))$$

$$Y^2 = \frac{16}{9} \left(1 - \frac{X^2}{25}\right) \rightarrow \frac{9Y^2}{16} = 1 - \frac{X^2}{25}$$

$$\frac{9Y^2}{16} + \frac{X^2}{25} = 1 \quad +1 \text{ for answer}$$

2. A fire-breathing duck is moving along a curve so that its position at time t is $(\cos(2t), \frac{1}{2}t^2)$.

Find the distance of the particle traveled from $t=2$ to $t=4$.

$$X = \cos(2t) \quad \frac{dx}{dt} = -2 \sin(2t)$$

$$Y = \frac{1}{2}t^2 \quad \frac{dy}{dt} = t$$

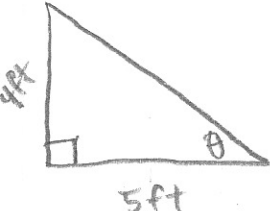
$$L = \int_{t_0}^t \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt$$

$$L = \int_2^4 \sqrt{t^2 + (-2 \sin(2t))^2} dt = 6.752$$

+1 for integral

+1 for answer

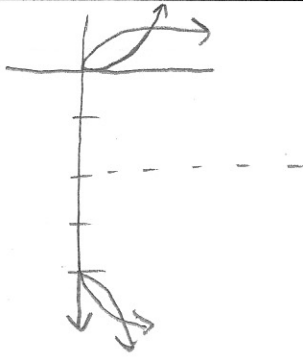
3. Szamos' child is crawling away from him at a rate of 2ft/sec. Szamos is 4 ft tall and standing still. When the child is 5 ft away, how fast is the angle of elevation from the child to Szamos changing?



$\theta = \tan^{-1}(y/x)$ $\frac{dy}{dt} = 0$ $\frac{dx}{dt} = 2\text{ft/sec}$
 $\tan\theta \cdot x = y$

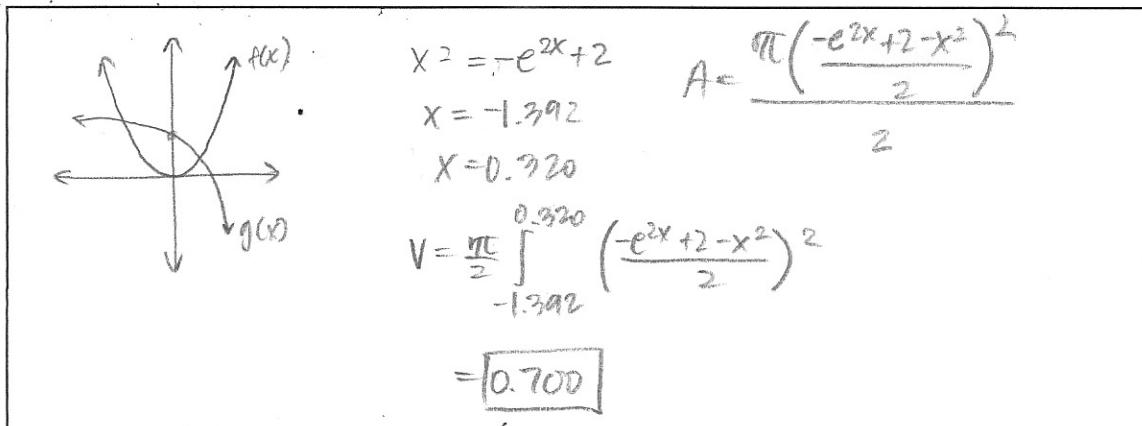
$\theta = \tan^{-1}(y/x)$ $\frac{dy}{dt} = \tan\theta \frac{dx}{dt} + x \sec^2\theta \frac{d\theta}{dt}$ +1 for derived equation
 $\theta = \tan^{-1}(4/5)$ $0 = \tan(.675)(2\text{ft/sec}) + (5\text{ft})(\sec^2(.675)) \frac{d\theta}{dt}$
 $\theta = .675$ $\frac{d\theta}{dt} = -.195 \text{ radians}$ +1 for answer

4. Find the volume of a shape enclosed by the functions $f(x)=x^2$ and $g(x)=\sqrt{x}$ rotated around $y=-2$.

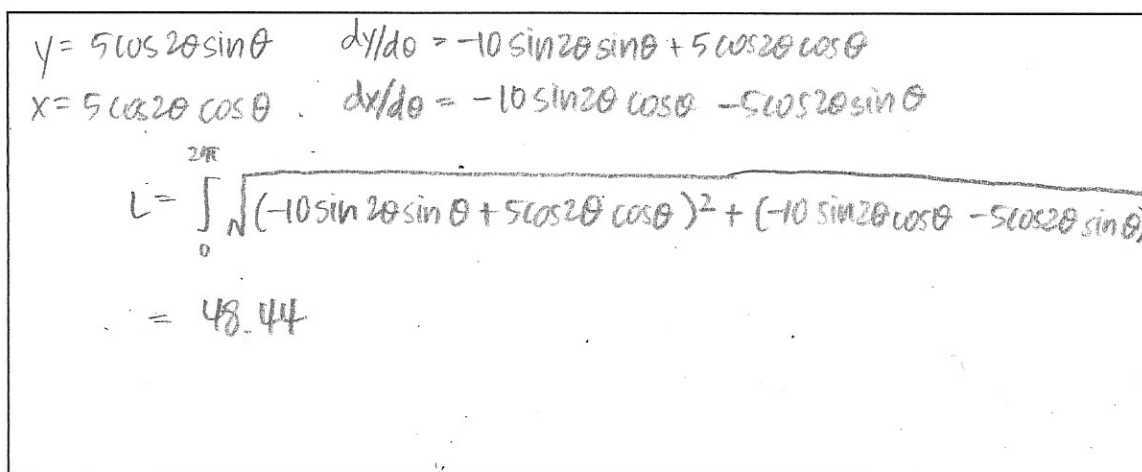


$A = \pi r_0^2 - \pi r_1^2$
 $= \pi (r_0^2 - r_1^2)$
 $A = \pi [(\sqrt{x} + 2)^2 - (x^2 + 2)^2]$
 $V = \pi \int_0^1 [(\sqrt{x} + 2)^2 - (x^2 + 2)^2] dx$
 $V = 5.131$

5. Find the volume of a shape enclosed by $f(x)=x^2$ and $g(x)=-e^{2x}+2$ with semicircles as cross sections.



6. Find the length of the polar curve $r = 5 \cos(2\theta)$ on $[0, 2\pi]$.



7. Find the area of the shape enclosed by the circle $r=3$ and the limaçon $r=3 - \cos(2\theta)$ in the first quadrant.

