Answer Key

1. **Parametric Functions:**

$y^2 = (4)^2(sin(t))^2 \qquad x^2 = (3)^2(cos(t))^2$

$\frac{y^2}{16} = sin^2(t) \qquad \frac{x^2}{9} = cos^2(t)$

$\frac{y^2}{16} + \frac{x^2}{9} = sin^2(t) + cos^2(t)$

Answer: $\frac{y^2}{16} + \frac{x^2}{9} = 1$

2. **Parametric (Vector) Motion:**

a) x-coordinate:

$$x(5) = 3 + \int_1^5 \sin(t)\, dt$$

Answer: $x(5) = 3.257$ (or 3.256)

b) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2^t}{sin(t)} = \frac{2^{(5)}}{sin(5)} = -33.371$ (or -33.370)

c) Distance:

$$D = \int_3^4 \sqrt{(sin(t))^2 + (2^t)^2}\, dt$$

Answer: $D = 11.548$ or (11.547)

3. **Related Rates:**

a)

$\frac{r_1}{h_1} = \frac{r_2}{h_2}$

$\frac{r_1}{h_1} = \frac{5}{10}$; where r = radius and h = height

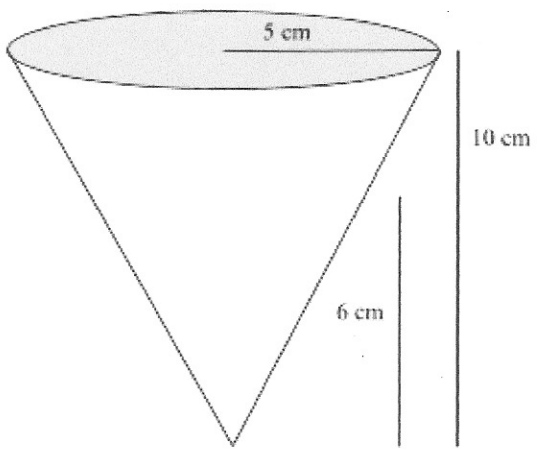
$\frac{r_2}{6} = \frac{5}{10}$

$r_2 = 3\ cm$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(3\ cm)^2(5\ cm)$

Answer: $V = 47.124\ cm^3$



b)

$\frac{r_1}{h_1} = \frac{5}{10} = \frac{1}{2}$

$r_1 = \frac{1}{2}h_1$

$V = \frac{1}{3}\pi(\frac{1}{2}h_1)^2(h)$

$V = \frac{1}{12}\pi {h_1}^3$

$\frac{dV}{dt} = \frac{3}{12}\pi {h_1}^2$

$\frac{dV}{dt} = 3\ cm^3/min - 1\ cm^3/min = 2\ cm^3/min$

$2 = \frac{1}{4}\pi(6)^2 \frac{dh}{dt}$

Answer: $\frac{dh}{dt} = 0.071\ cm/min$

4. **Volumes of Rotation:**

Points of Intersection:

$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0\ ;\ x = 1$

Area:

$A = \pi(\sqrt{x})^2 - \pi(x^{2})^{2}$

$A = \pi x - \pi x^4$

Volume:

$V = \int_0^1 (\pi x - \pi x^4)\, dx$

$V = \frac{\pi}{2}x^2 - \frac{\pi}{5}x^5 + C\ \{0 \le x \le 1\}$

$V = (\frac{\pi}{2} - \frac{\pi}{5}) - 0$

Answer: $V = \frac{3\pi}{10}\ units^3$

5. **Volumes on a Base:**

Points of intersection:

$2\sqrt{x} = \frac{x^3}{20}$

Or graph the two functions on calculator

x = 0, 4.373

Volume:

$V = \frac{\pi}{2} \int_0^{4.373} (\frac{1}{2}(2\sqrt{x} - \frac{x^3}{20}))^2\, dx$

$V = \frac{\pi}{8} \int_0^{4.373} (2\sqrt{x} - \frac{x^3}{20})^2 dx$

Answer: $V = 5.961$ (*or* 5.960)

6. **Polar Curves (Conversion, Slopes, Lengths):**

a) $r = 5\ cos(3\theta)$

$x = r\ cos(\theta)$

$x = 5\ cos(3\theta)\ cos(\theta)$

$\frac{dx}{d\theta} = -15\ cos(\theta)\ sin(3\theta) - 5\ sin(\theta)\ cos(3\theta)$

$y = r\ sin(\theta)$

$y = 5\ cos(3\theta)\ sin(\theta)$

$\frac{dy}{d\theta} = 5\ cos(\theta)\ cos(3\theta) - 15\ sin(\theta)\ sin(3\theta)$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$$\frac{dy}{dx} = \frac{5\cos(\theta)\cos(3\theta) - 15\sin(\theta)\sin(3\theta)}{-15\cos(\theta)\sin(3\theta) - 5\sin(\theta)\cos(3\theta)}$$

$$\frac{dy}{dx} = \frac{5\cos(\frac{\pi}{3})\cos(\pi) - 15\sin(\frac{\pi}{3})\sin(\pi)}{-15\cos(\frac{\pi}{3})\sin(\pi) - 5\sin(\frac{\pi}{3})\cos(\pi)}$$

Answer: $\frac{dy}{dx} = -0.577\ (or\ -\sqrt{3})$

b) $x = 5\cos(3\theta)\cos(\theta)$

$x = 5\cos(\pi)\cos(\frac{\pi}{3})$

$x = -2.500\ (or\ \frac{-5}{2})$

$y = 5\cos(3\theta)\sin(\theta)$

$y = 5\cos(\pi)\sin(\frac{\pi}{3})$

$y = -4.330\ (or\ \frac{-5\sqrt{3}}{2})$

$y = mx + b$

$(\frac{-5\sqrt{3}}{2}) = (-\sqrt{3})(\frac{-5}{2}) + b$

$b = -5\sqrt{3}$

Answer: $y = -x\sqrt{3} - 5\sqrt{3}$

7. **Polar Curves (Areas):**

a) Points of intersection:

$4 = 4 - 4\sin(2\theta)$

$0 = -4\sin(2\theta)$

$0 = \sin(2\theta)$

$\theta = 0,\ \frac{\pi}{2},\ \pi$

Area:

$A = A_1 + A_2$

$A_1 = \pi r^2 / 4$

$A_1 = 4\pi$

$$A_2 = \frac{1}{2}\int_0^{\frac{\pi}{2}} (4 - 4\sin(2\theta))^2\, d\theta$$

$$A = 4\pi + \frac{1}{2}\int_0^{\frac{\pi}{2}} (4 - 4\sin(2\theta))^2\, d\theta$$

Answer: $A = 15.416\ (or\ 15.415)$

b) $x = r\cos(\theta)$

$x = (4 - 4\sin(2\theta))\cos(\theta)$

$\frac{dx}{d\theta} = -\sin(\theta)(4 - 4\sin(2\theta)) - 8\cos(\theta)\cos(2\theta)$

$\frac{dx}{d\theta} = -\sin(\frac{\pi}{6})(4 - 4\sin(\frac{\pi}{3})) - 8\cos(\frac{\pi}{6})\cos(\frac{\pi}{3})$

Answer: $\frac{dx}{d\theta} = -3.732\ (or\ -3.731)$

c) $\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{dr}{d\theta} \cdot 2$

$r = (4 - 4\sin(2\theta))$

$\frac{dr}{d\theta} = -8\cos(2\theta)$

$\frac{dr}{d\theta} = -8\cos(\frac{\pi}{3})$

$\frac{dr}{d\theta} = -4$

$\frac{dr}{dt} = (-4)(2)$

Answer: $\frac{dr}{dt} = -8$.