<Answer key> P.3 Lina Lee & Justin Haw

1. (0, 6)

2. $m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos(t^2)}{2\sin(t)}$ at $t = 1$

≈ 0.321

$y = 0.321x + 5.037$.

3.

$\frac{dA}{dt} = 108\pi \ m^2/s$ $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$108\pi = 2\pi(6)\frac{dr}{dt}$

$\frac{108\pi}{12\pi} = \frac{dr}{dt} =$ 9. m/s

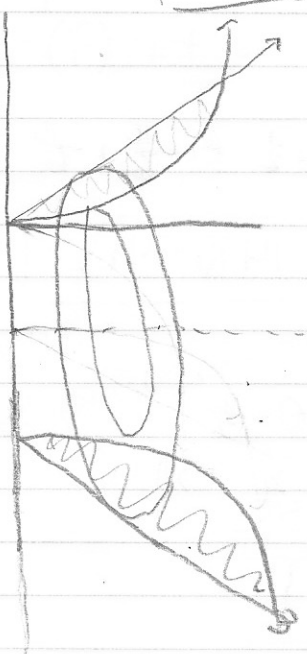
Answer Key

4. a. $x = \frac{1}{4}x^2$ $\frac{1}{x} = \frac{1}{4}$ $x = 4$ $\frac{1}{4}(4)^2 = 4$ $(4,4) \rightarrow$ intersection point

$$\int_0^4 x - \frac{1}{4}x^2\,dx \rightarrow \frac{1}{2}x^2 - \frac{1}{12}x^3 \Big|_0^4 \quad (8 - \tfrac{16}{3}) - (0) \rightarrow (\tfrac{24}{3} - \tfrac{16}{3}) = \frac{8}{3}$$

$$\boxed{A = \frac{8}{3}}$$

b.



$A = \pi r^2$ $r = x + 2$ $A = \pi(x^2 + 4x + 4)$

$(x+2)(x+2) = x^2 + 4x + 4$

$A = \pi r^2$ $r = \frac{1}{4}x^2 + 2$ $A = \pi(\frac{1}{16}x^4 + \frac{1}{2}x^2 + 4)$

$(\frac{1}{4}x^2 + 2)(\frac{1}{4}x^2 + 2) = \frac{1}{16}x^4 + \frac{1}{2}x^2 + 4$

$$\pi \int_0^4 (x^2 + 4x + 4) - (\tfrac{1}{16}x^4 + \tfrac{1}{2}x^2 + 4)\,dx$$

$$\pi\left((\tfrac{1}{3}x^3 + 2x^2 + 4x) - (\tfrac{1}{80}x^5 + \tfrac{1}{6}x^3 + 4x)\right)\Big|_0^4$$

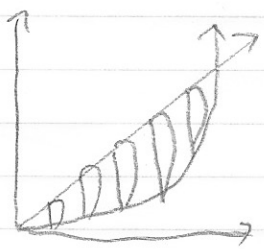
$$\pi\left((\tfrac{64}{3} + 32 + 16) - (\tfrac{64}{5} + \tfrac{32}{3} + 16)\right)$$

$$\pi\left((\tfrac{64}{3} + \tfrac{96}{3} + \tfrac{48}{3}) - (\tfrac{192}{15} + \tfrac{160}{15} + \tfrac{240}{15})\right)$$

$$\pi(\tfrac{208}{3} - \tfrac{592}{15}) \rightarrow \pi(\tfrac{1040}{15} - \tfrac{592}{15})$$

$$\boxed{V = \frac{448\pi}{15}}$$

5.



$A = \frac{1}{2}\pi r^2$ $r = \frac{1}{4}x^2$ $A = \frac{1}{2}\pi(\frac{1}{16}x^4)$

$A = \frac{1}{32}\pi x^4$

$$\frac{1}{32}\pi \int_0^4 x^4\,dx \qquad \frac{1}{32}\pi\left(\frac{1}{5}x^5\right)\Big|_0^4$$

$$\frac{1}{32}\pi\left(\frac{1024}{5}\right) \leftarrow \frac{1024\pi}{160} \leftarrow \boxed{\frac{32\pi}{5}}$$

6. a. $x = r\cos\theta$ $\quad x = -3\cos 135 \rightarrow \frac{3\sqrt{2}}{2}$

$y = r\sin\theta$ $\quad y = -3\sin 135 \rightarrow \frac{-3\sqrt{2}}{2}$ $\quad \boxed{\left(\frac{3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)}$

b. $r = \sqrt{x^2 + y^2}$ $\quad r = \sqrt{(-3)^2 + (5)^2} \rightarrow r = \sqrt{34}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ $\quad \tan\left(\frac{5}{-3}\right)$ $\quad \theta = -59°$ $\quad \boxed{(\sqrt{34}, -59°)}$

c. $(r\cos\theta)^2 + 2(r\cos\theta)$ $\quad y = x^2 + 2x$ $\quad (1, 3)$ $\quad r = \sqrt{x^2 + y^2}$ $\quad r = \sqrt{(1)^2 + (3)^2}$

$r^2\cos^2\theta + 2r\cos\theta$ $\quad r = \sqrt{10}$

$\boxed{10\cos^2 71.6° + 2\sqrt{10}\cos 71.6°}$ $\quad \tan\left(\frac{3}{1}\right) = 71.6°$

7. a. $3 + 4\sin\theta = 5$ $\quad 4\sin\theta = 2 \rightarrow \sin\theta = \frac{1}{2}$ $\quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (3 + 4\sin\theta)^2 - 25 \, d\theta \qquad \boxed{A = 14\pi + 14\sqrt{3}}$$

b. $3 + 4\sin\theta = 7$ $\quad 4\sin\theta = 4$ $\quad \sin\theta = 1$ $\quad \theta = \frac{\pi}{2}$ $\quad 311.4° \cdot \frac{\pi}{180}$

$3 + 4\sin\theta = 0$ $\quad 4\sin\theta = -3$ $\quad \sin\theta = \frac{-3}{4}$ $\quad -311.4°$

$\theta = 1.73\pi$

$$\frac{1}{2}\int_{\frac{\pi}{2}}^{1.73\pi} (3 + 4\sin\theta)^2 \, d\theta \qquad \boxed{A = 26.9}$$