Answer Key (by Hannah Chew and Taleen Sarkissian)

1) $x = 2\cos t \qquad y = 5\sin t \qquad \frac{x^2}{4} + \frac{y^2}{25} = \cos^2 t + \sin^2 t$

$x^2 = 4\cos^2 t \qquad y^2 = 25\sin^2 t$

$\cos^2 t = \frac{x^2}{4} \qquad \sin^2 t = \frac{y^2}{25}$

$$\boxed{\frac{x^2}{4} + \frac{y^2}{25} = 1}$$

2) $X(t) \ \langle 6t^2 + 2, \frac{1}{2}t^4 \rangle$

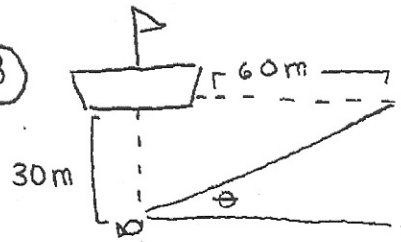
$V(t) = X'(t) \ \langle 12t, 2t^3 \rangle$

$V(2) = X'(2) \ \langle 24, 16 \rangle$

$R = \sqrt{(V_x)^2 + (V_y)^2}$

$= \sqrt{24^2 + 16^2}$

$= \boxed{28.84 \text{ m/s}}$

3) 

$\theta = \tan^{-1}\left(\frac{30}{60}\right) = .4636 \text{ rad}$

$\tan\theta = \frac{y}{x}$

$y = x\tan\theta$

$\frac{dy}{dt} = \frac{dx}{dt}(\tan\theta) + x\sec^2\theta\frac{d\theta}{dt}$

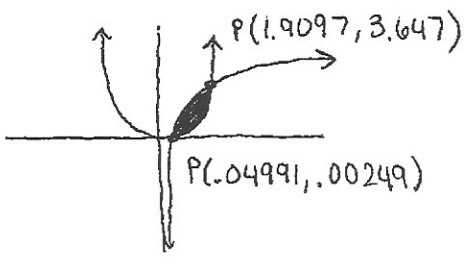
$0 = 5(\tan .4636) + 60\sec^2(.4636)\left(\frac{d\theta}{dt}\right)$

$-2.5 = 75\frac{d\theta}{dt} \rightarrow \boxed{\frac{d\theta}{dt} = -.033 \text{ rad/min}}$

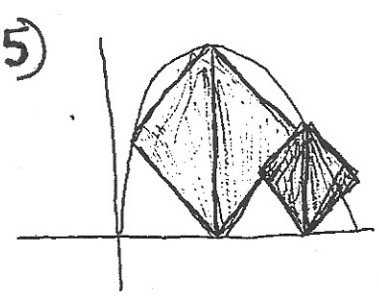
4) $A = \pi(r_1^2 - r_2^2)$

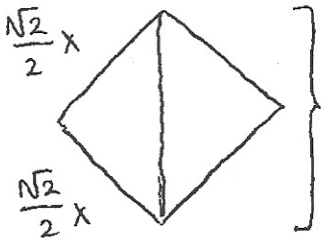
$= \pi((\ln x + 3)^2 - (x^2)^2)$

$V = \pi\int_{.04991}^{1.9097}[(\ln x + 3)^2 - x^4]\,dx = \boxed{31.762}$



5)





$$A = \left[\frac{\sqrt{2}}{2} f(x)\right]^2$$

$$= \frac{1}{2}\sin^2 x$$

$$V = \frac{1}{2}\int_0^{\pi} \sin^2 x \, dx = \boxed{\frac{\pi}{4} = .7854}$$

6) $x = (3+4\sin\theta)\cos\theta = 3\cos\theta + 2\sin(2\theta)$

$$\frac{dx}{d\theta} = -3\sin\theta + 2\cos(2\theta)(2)$$

$$= -3\sin\theta + 4\cos(2\theta)$$

$y = 3\sin\theta + 4\sin^2\theta$

$$\frac{dy}{d\theta} = 3\cos\theta + 8\sin\theta\cos\theta = 3\cos\theta + 4\sin 2\theta$$

$$L = \int_0^{2\pi} \sqrt{\left[-3\sin\theta + 4\cos(2\theta)\right]^2 + \left[3\cos\theta + 4\sin 2\theta\right]^2}\, d\theta$$

$$= \boxed{28.814}$$

b) $$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3\cos(\pi) + 4\sin(2\pi)}{-3\sin(\pi) + 4\cos(2\pi)} = \boxed{\frac{-3}{4}}$$