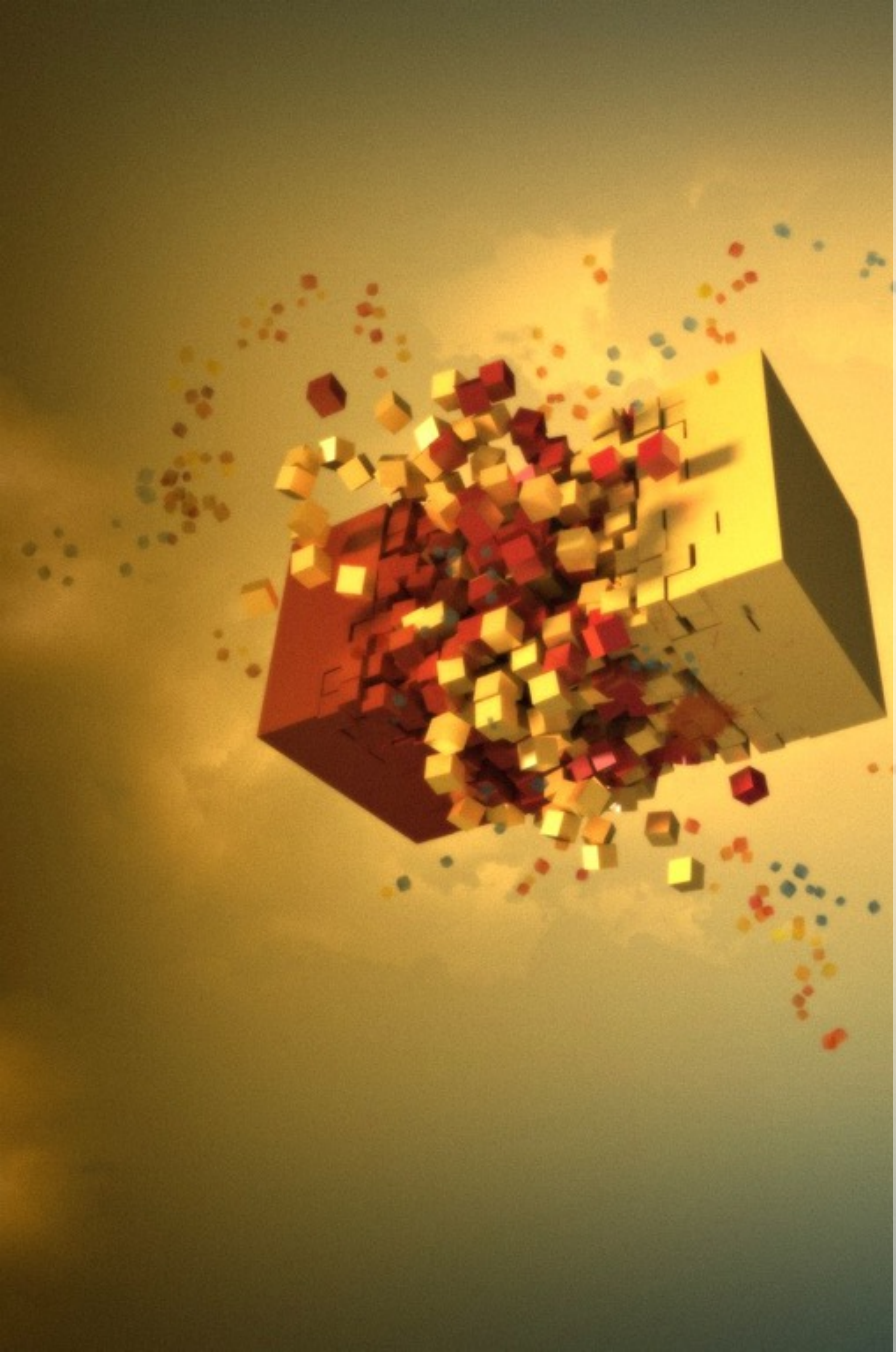
MOMENTUM

General Physics

"The world is wide, and I will not waste my life in friction when it could be turned into momentum." — Frances E. Willard

How hard would a puck have to be shot to be able to knock the goalie himself backward into the net?



CONSERVATION OF…

- Energy
- Linear Momentum
- Angular Momentum
- Electric Charge

MOMENTUM

- What is it?
- It's another way to think about objects in motion
  - $\mathbf{p} = m\mathbf{v}$
- Momentum is…
  - a vector
  - dependent on your reference frame
  - measured in kg•m/s

DO PROBLEMS #1–5

MOMENTUM

- A fast-moving car has more momentum than a slow-moving car with the same mass
- A heavy truck has more momentum than a small car at the same speed
- The more momentum an object has…
    - the harder it is to stop it
    - the greater effect it will have if brought to rest by impact or collision

MOMENTUM & FORCE

- A net force F_{net} accelerates a car up to some speed over the course of time Δt.
- What is the change in momentum of the car?
  - $\Delta \mathbf{p} = F_{net}\Delta t$
  - Change in momentum called impulse (J)
  - The impulse equation is equivalent to Newton's 2nd Law

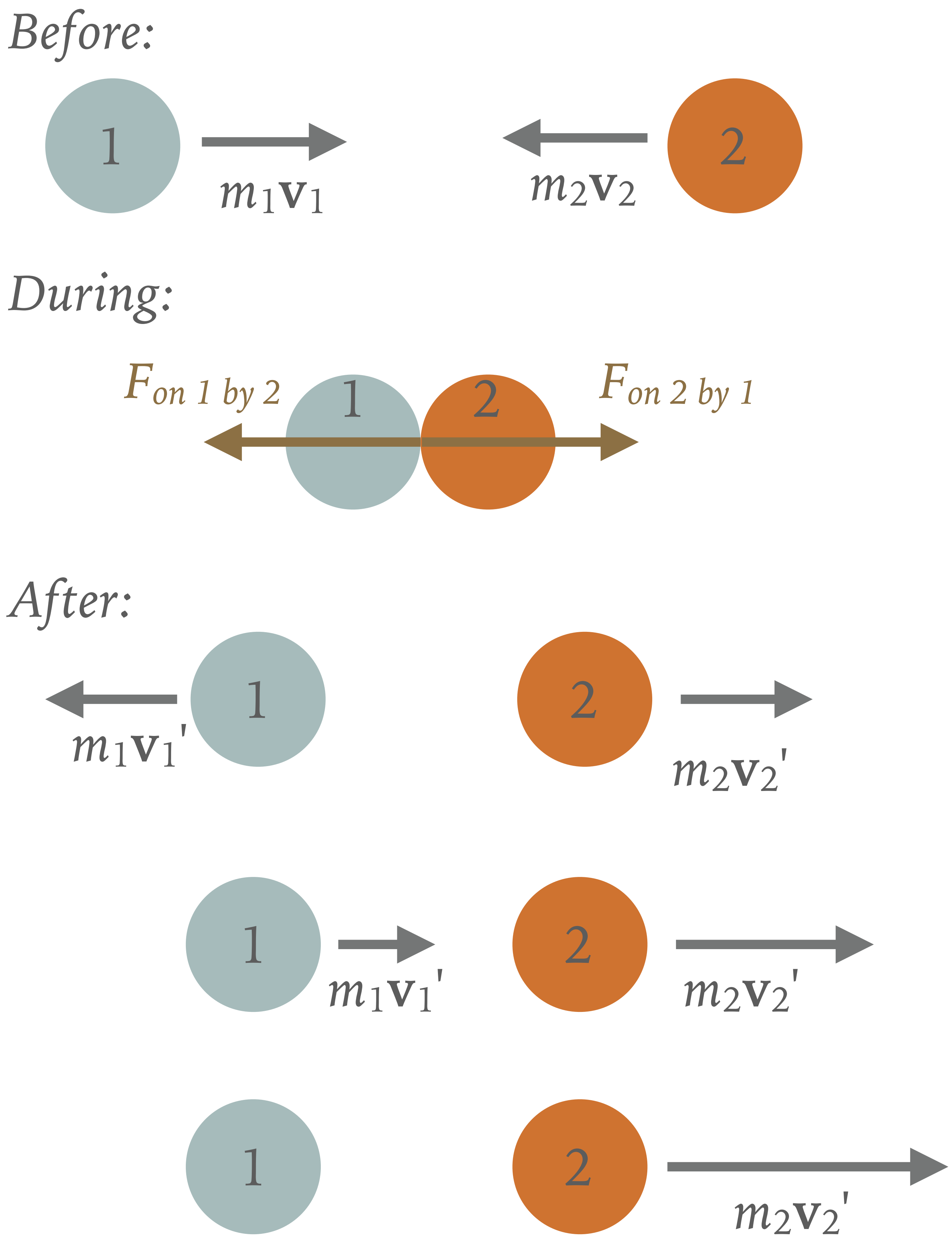
EXAMPLE 1

- Mario is cleaning up the town with his Flash Liquidizer Ultra Dousing Device, aka FLUDD.
- Water leaves the FLUDD at a rate of 19 kg/s with a speed of 37 m/s and is aimed at a graffitied wall, which stops it (that is, we ignore any splashing back).
- What is the force exerted by the water on the wall?
  - *Answer*: $F = 703\ \text{N}$

SANITY CHECK

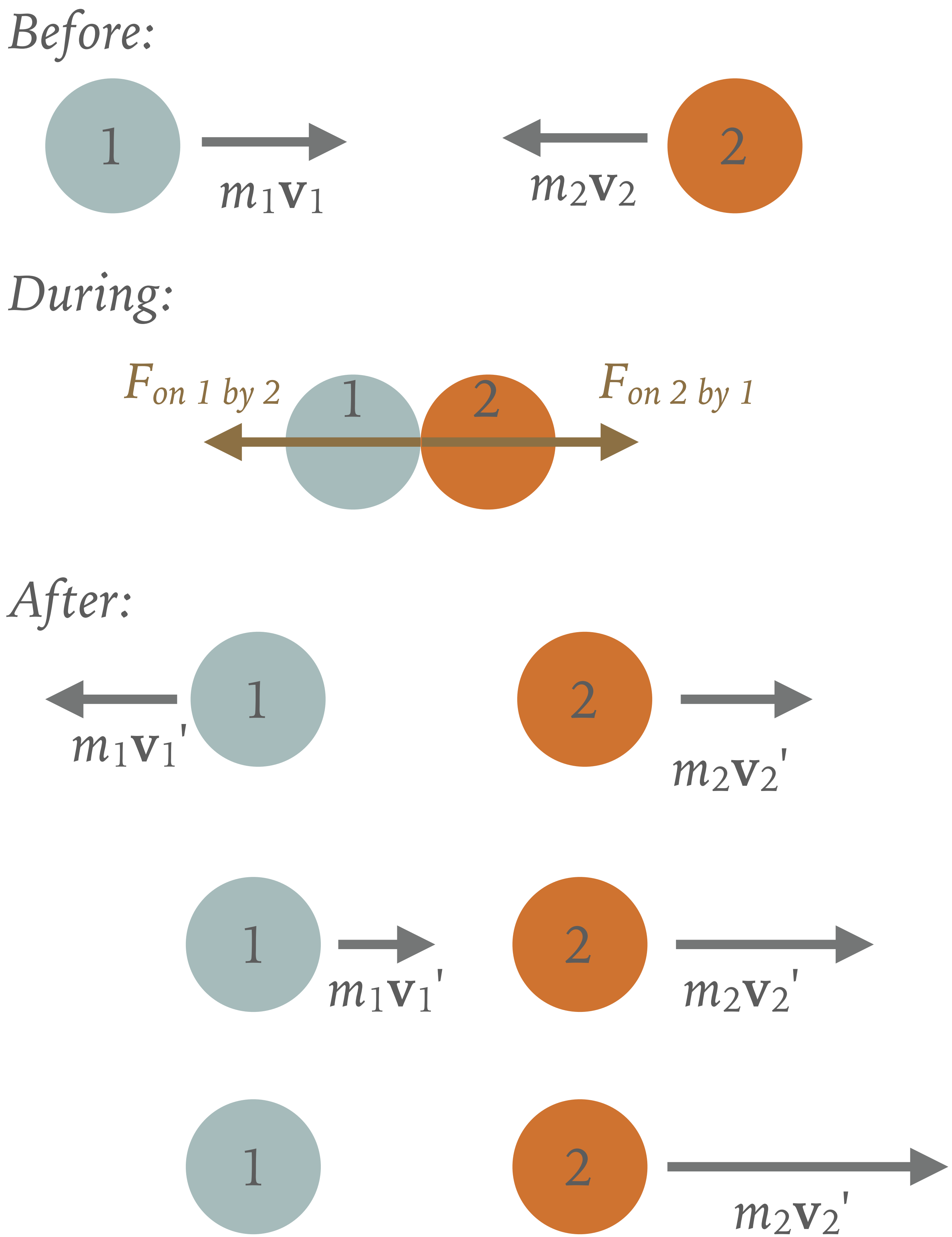
- What if the water *does* splash from the wall? Would the force on the wall be greater, less, or the same?
  - *Ans. Greater*

DO PROBLEMS #6–10



CONSERVATION OF MOMENTUM

- Picture two billiard balls set for a head-on collision
  - (ignore any retarding forces on the billiard balls)
- After the collision, the momentum of each of the two balls will change
  - But the *sum* of the momenta will be the same after the collision as it was before



CONSERVATION OF MOMENTUM

- momentum before = momentum after
- $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1' + m_2\mathbf{v}_2'$
- The total vector momentum of the two ball system is conserved
- Conservation of Momentum was discovered experimentally, but can be derived using Newton's 2nd Law

THE LAW CONSERVATION OF MOMENTUM

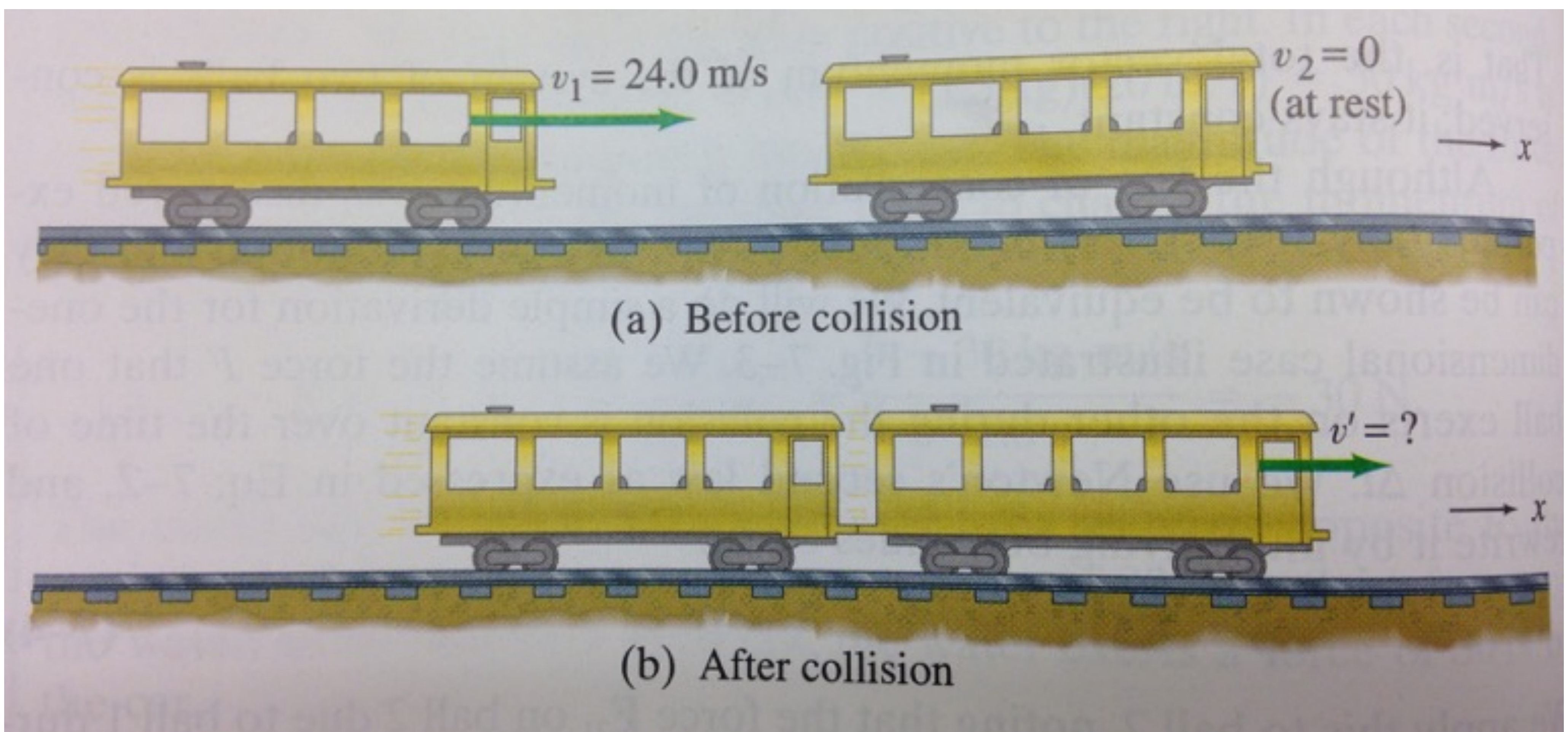
- *The total momentum of an isolated system remains constant*
  - System — a set of objects that interact with each other
  - Isolated system — system in which the only forces present are those between the objects of the system

SANITY CHECK

- You drop a rock and allow it to plummet in free fall
- Is momentum conserved?
  - *Ans. It is if you include you the Earth in your system*

EXAMPLE 2

- A 10,000 kg railroad car traveling at a speed of 24.0 m/s strikes an identical car at rest.
- If the cars lock together as a result of the collision, what is their common speed afterwards?
  - *Answer*: $v_f = 12.0$ m/s



CONSERVATION OF MOMENTUM

- Also useful for evaluating certain types of explosions
- E.g. rocket propulsion
  - Before the rockets fire, the total momentum of the system (rocket + fuel) is zero
  - Backward momentum of the expelled exhaust = forward momentum of the rocket
- Other examples: recoil of a gun or throwing a package from a boat

EXAMPLE 3

- Calculate the recoil velocity of a 5.0-kg rifle that shoots a 0.050-kg bullet at a speed of 120 m/s
  - *Answer*: $v_R' = -1.2$ m/s

IMPULSE

- Conservation of Momentum is a useful tool for dealing with…
  - tennis racket or baseball bat striking a ball
  - two billiard balls colliding
  - one train car striking another
  - a hammer hitting a nail
  - collisions between atomic nuclei

IMPULSE

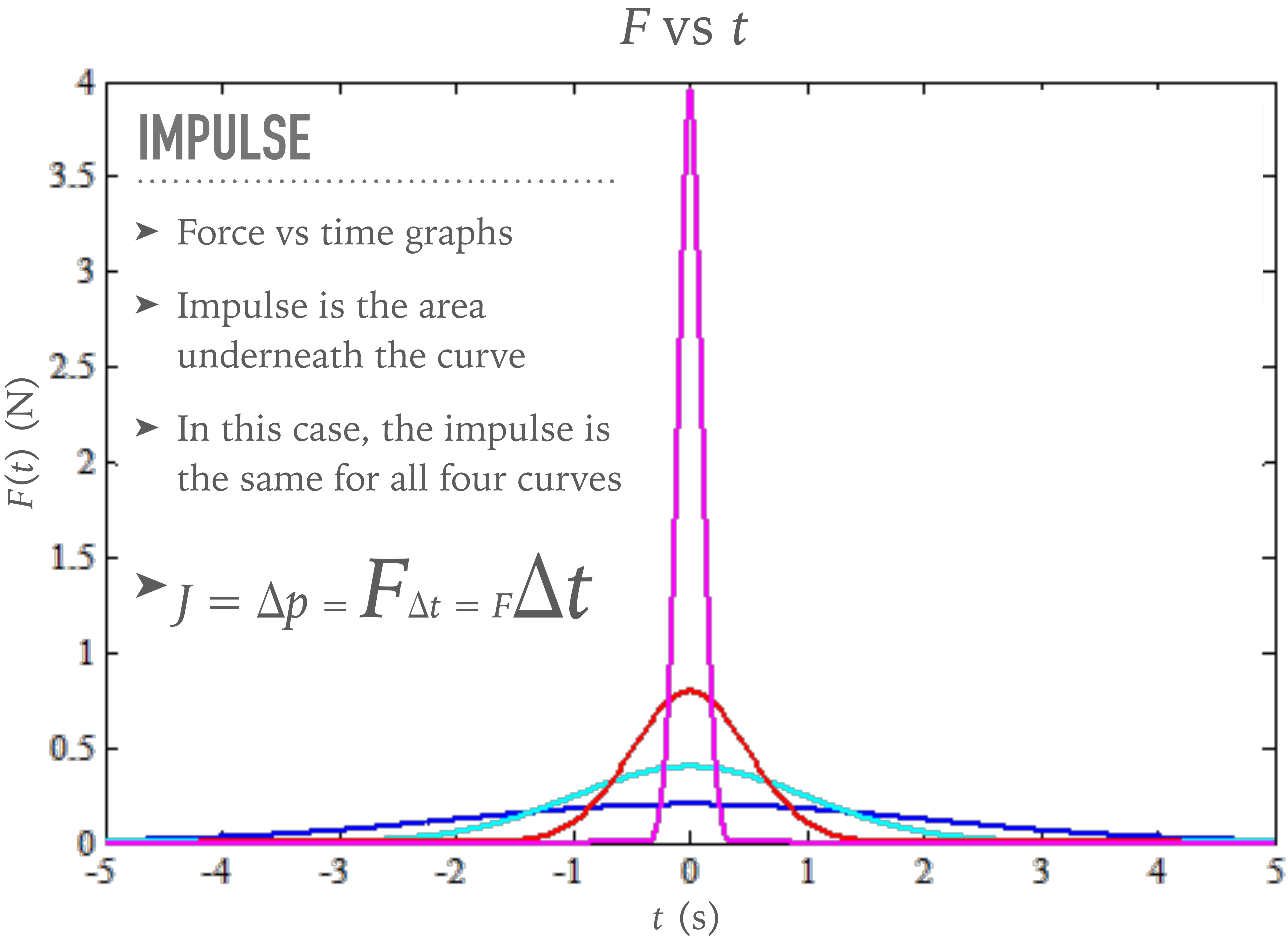
- When two objects collide, both objects deform
- Typically force jumps from zero to a very large value and back to zero in a short amount of time

https://www.youtube.com/watch?v=00I2uXDxbaE

https://www.youtube.com/watch?v=otHZwjElXwQ

IMPULSE

- Force vs time graphs
- Impulse is the area underneath the curve
- In this case, the impulse is the same for all four curves
- $J = \Delta p = F_{\Delta t} = F\Delta t$



EXAMPLE 4

a) With what speed would a 70-kg person hit the ground if they dropped from a height of 3.0 m?

   a) Answer: $v = 7.67$ m/s

b) Calculate the impulse experienced by this person when they land on the firm ground below.

   b) Answer: $J = -540$ N•s

- Would the average force exerted on the person's feet by the ground be larger if the person lands with stiff-legs or bent legs?

   c) Answer: stiff-legs

SANITY CHECK

- You’re charged with designing a car, and money is no object. In order to best keep passengers safe in the event of a collision, you should build your car out of
  - Aluminum
  - Diamond
- *Ans. Aluminum. Either way the impulse will be the same, but aluminum will spread a smaller force of impact over a greater time.*

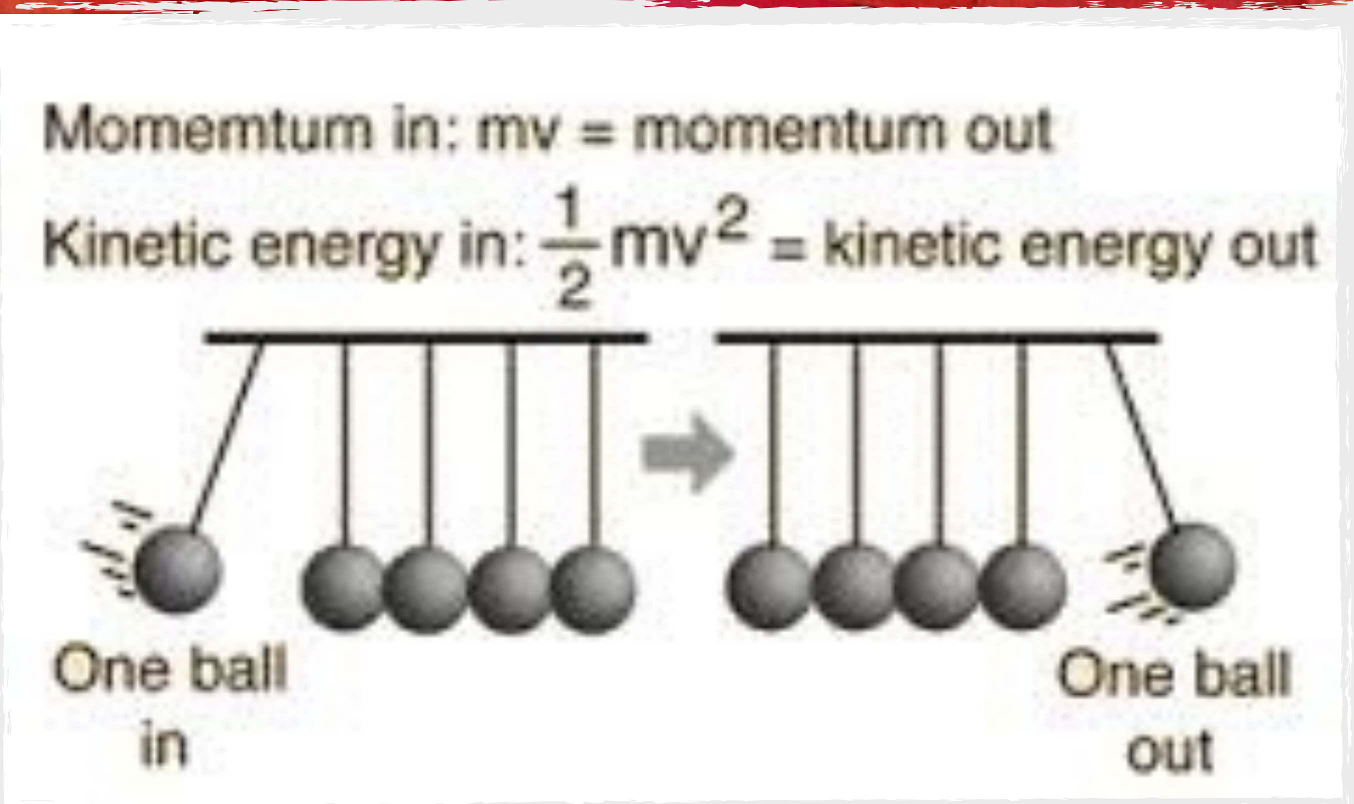
DO PROBLEMS #11–25

ELASTIC VS INELASTIC

- Elastic —

- Plastic —

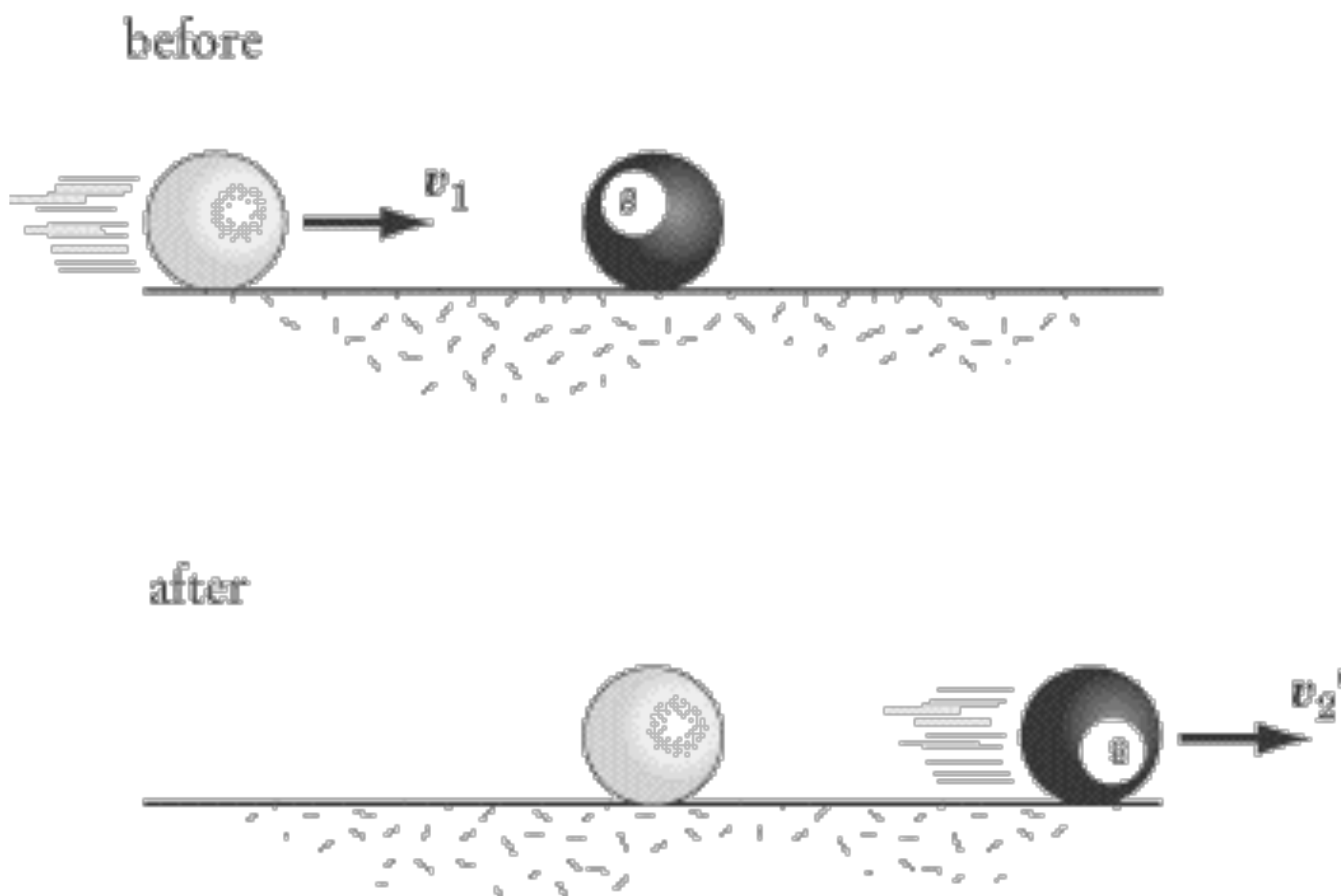
CONSERVATION OF ENERGY AND MOMENTUM IN COLLISIONS



- If the two objects in a collision are
  - very hard
  - and no heat is produced in the collision
- then kinetic energy is conserved as well

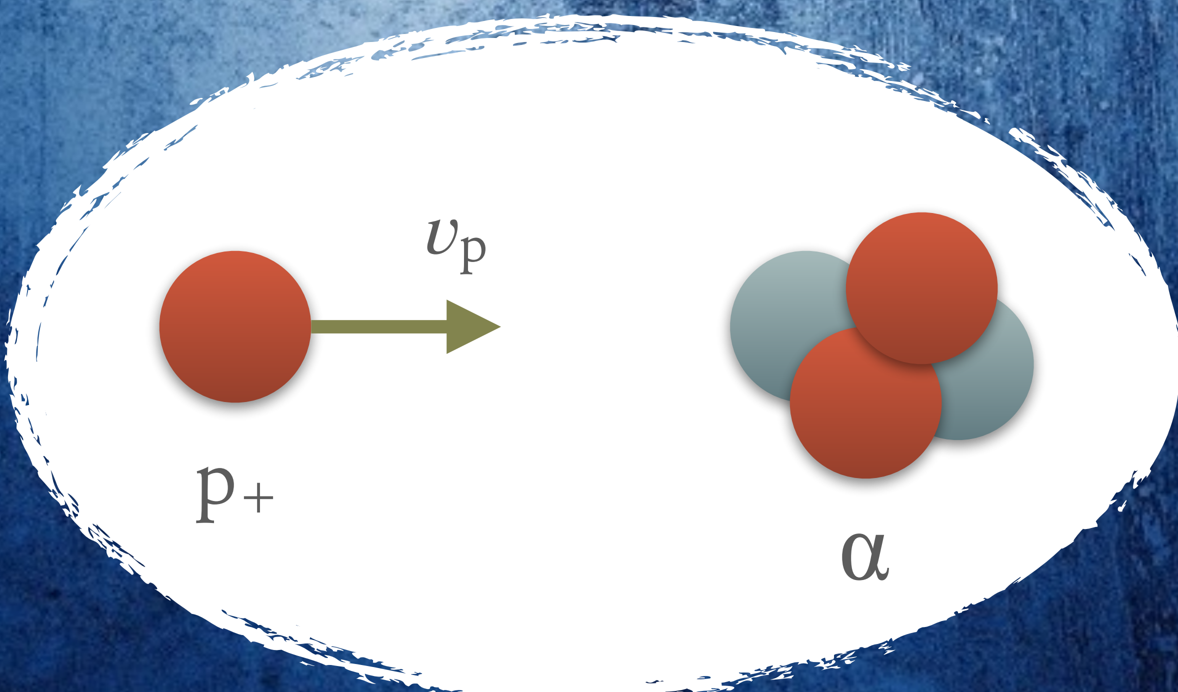
CONSERVATION OF ENERGY AND MOMENTUM IN COLLISIONS

- Collisions where total kinetic energy is conserved are called elastic collisions
  - $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$ [elastic collision]
- Perfect elastic collisions are an ideal which is never quite met
  - At least a little thermal energy is always lost



EXAMPLE 5

- A billiard ball of mass m moving with speed v collides head-on with a second ball of equal mass at rest. What are the speeds of the two balls after the collision, assuming it is elastic?
  - *Ans.* $v_1' = 0$, $v_2' = v$



EXAMPLE 6

- A proton of mass m_p = 1.67×10^{-27} kg traveling with a speed of 3.60×10^4 m/s has an elastic head-on collision with an alpha particle (a helium nucleus; $m_\alpha = 6.64\times10^{-27}$ kg) initially at rest.
- If the proton rebounds with a velocity of 2.15×10^4 m/s, how fast must the alpha particle be moving after the collision?
  - *Answer*: $v_\alpha' = 1.45\times10^4$ m/s

DO ELASTIC COLLISION PROBLEMS

INELASTIC COLLISIONS

- Collisions where kinetic energy is *not* conserved are called inelastic collisions
- Initial KE transformed to some other form (like thermal or potential) during collision
- In inelastic collisions, $KE_f < KE_i$
- Explosions are inelastic collision in reverse
  - PE → KE

INELASTIC COLLISIONS

- If the two objects stick together after the collision, it's called perfectly inelastic
  - E.g. football tackle
  - two balls of putty colliding
  - a bullet fired into a block of wood
- *Note: even though KE is not conserved in inelastic collisions, the total energy is always conserved, as is the total vector momentum*

EXAMPLE 7

- A 155 kg football player running at 6.00 m/s tackles his 103 kg opponent (initially at rest) in a perfectly inelastic collision.
- How fast do the two move after they collide?
  - *Answer*: $v_f = 3.60$ m/s
- How much of the initial kinetic energy is transformed into thermal or other forms of energy?
  - *Answer*: $\Delta KE = -1.12\ kJ$

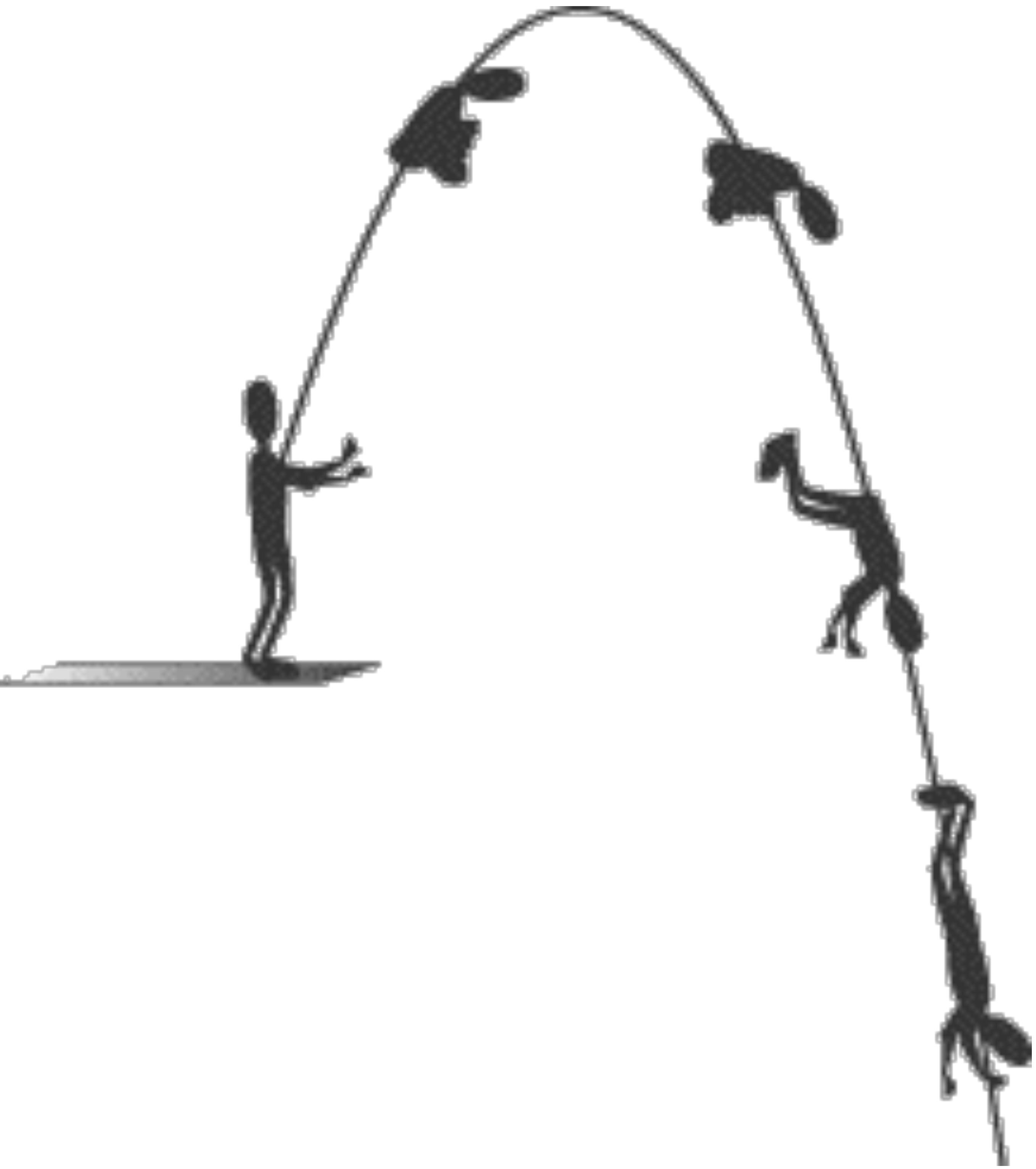
EXAMPLE 8

- Two spheres, both with mass m and speed v, collide head-on. Calculate the velocities after the collision assuming the collision is (a) perfectly elastic and (b) perfectly inelastic.

  a) $v_1' = -v$, $v_2' = +v$

  b) $v' = 0$

DO INELASTIC COLLISION PROBLEMS



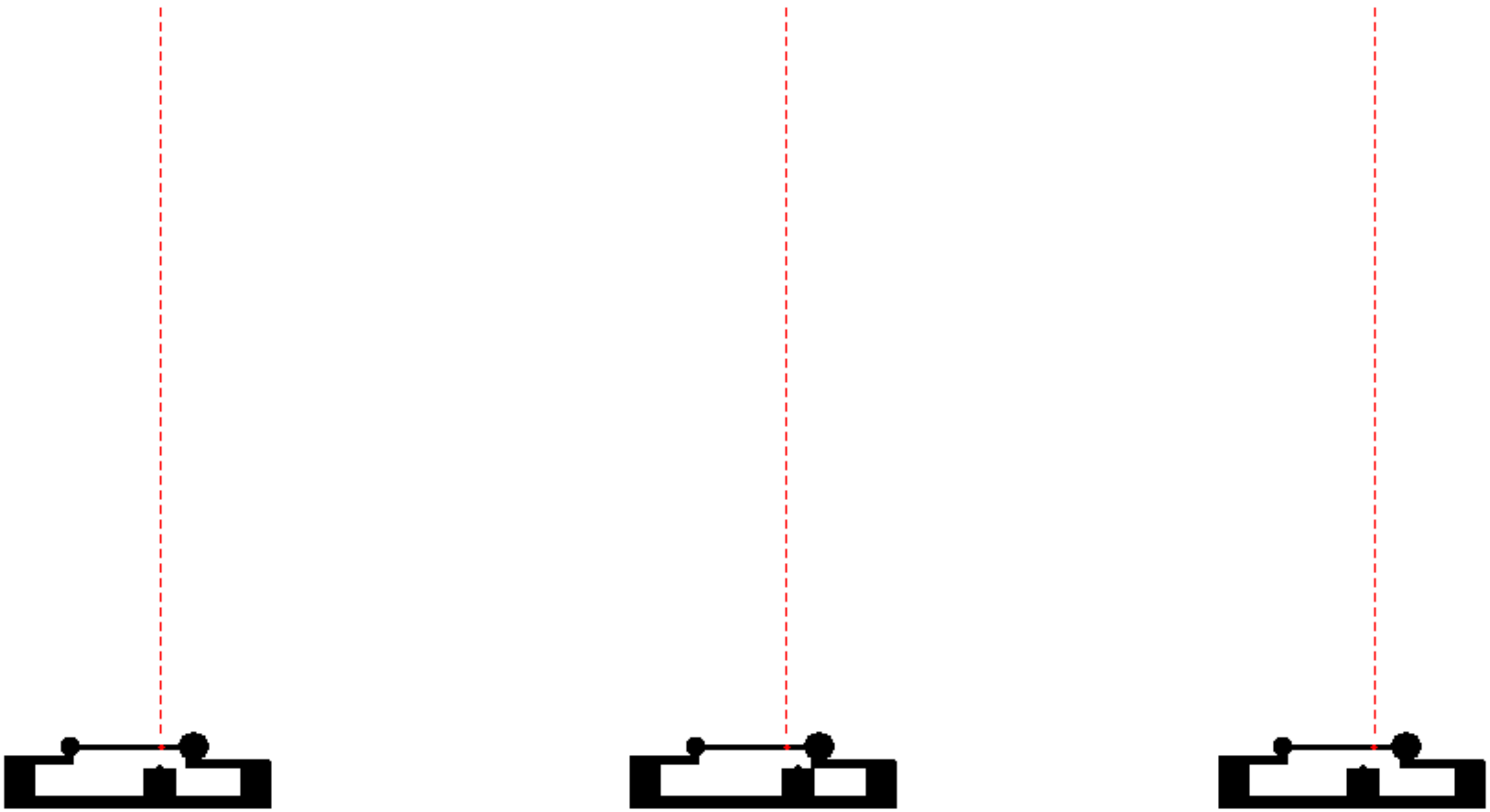
CENTER OF MASS

- In real life, objects aren't just points
- Real, extended bodies can undergo rotation, vibration, etc. in addition to translation
- E.g. the diver on the left experience parabolic translational motion as well as rotational motion
- Motion that is not pure translational motion is referred to as general motion

CENTER OF MASS

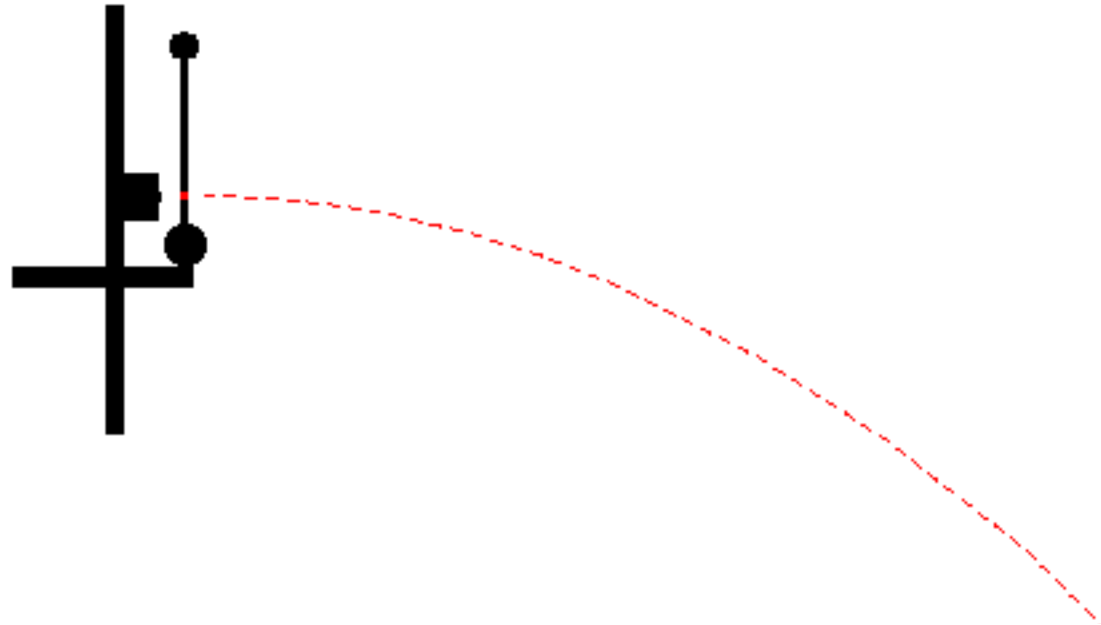
- Center of Mass (CM) of an object (or system of objects) is the unique point where…
  - all mass is considered to be "concentrated"
  - motion behaves as though all mass were converged into a point
  - a net force can be applied without causing the object to rotate
  - the object can be balanced

F on CM | *F* right of CM | *F* left of CM

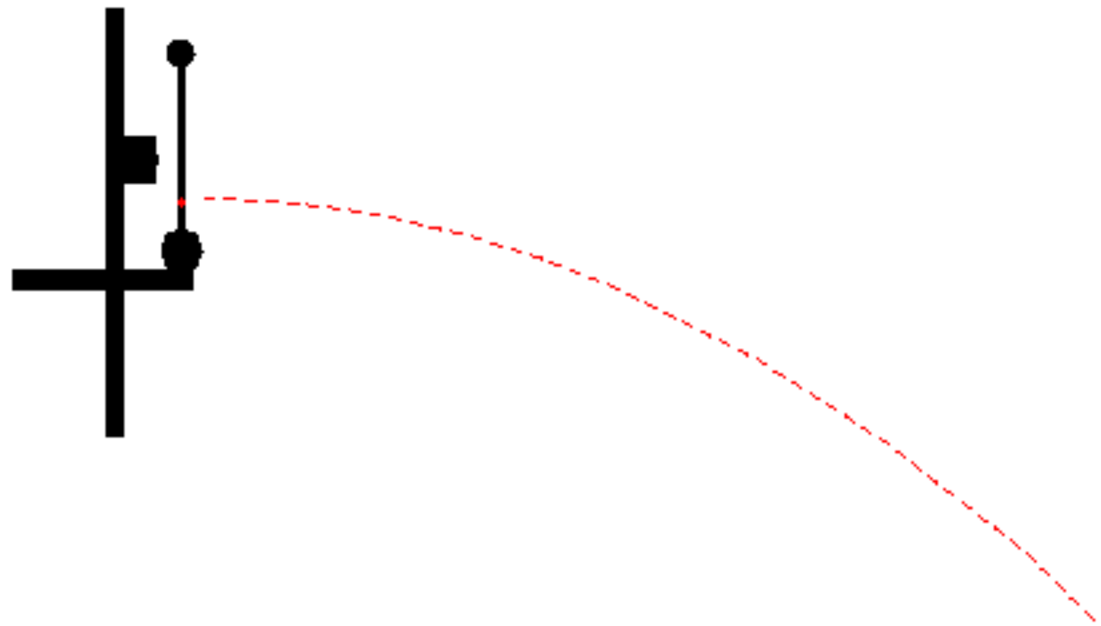


http://www.acs.psu.edu/drussell/Demos/COM/com-a.html

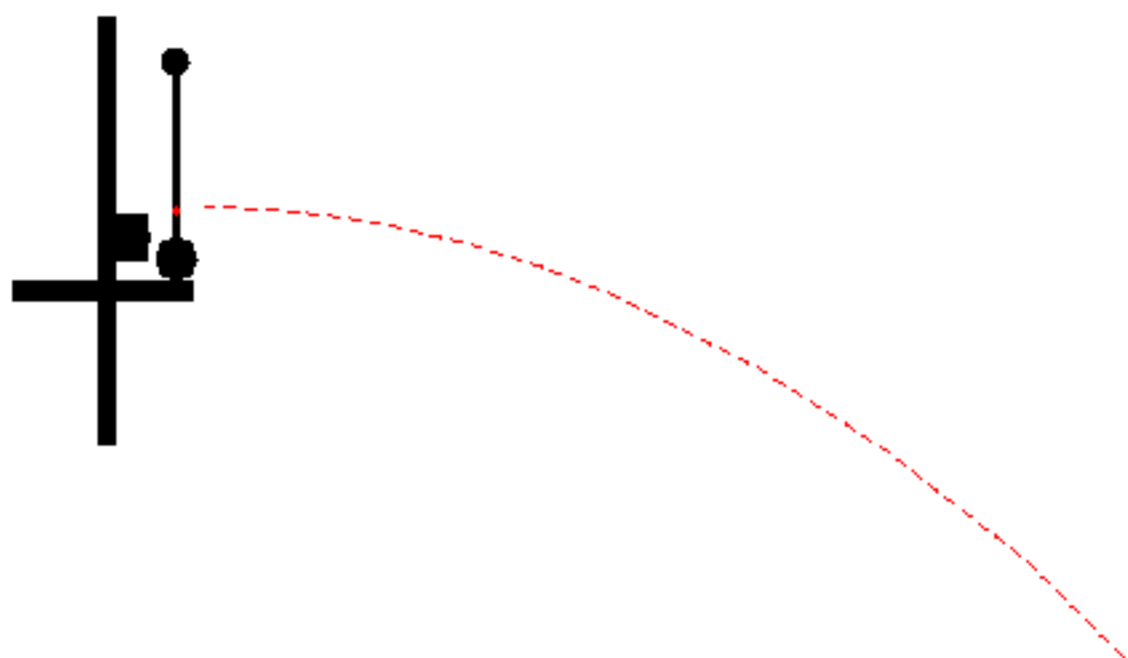
F on CM



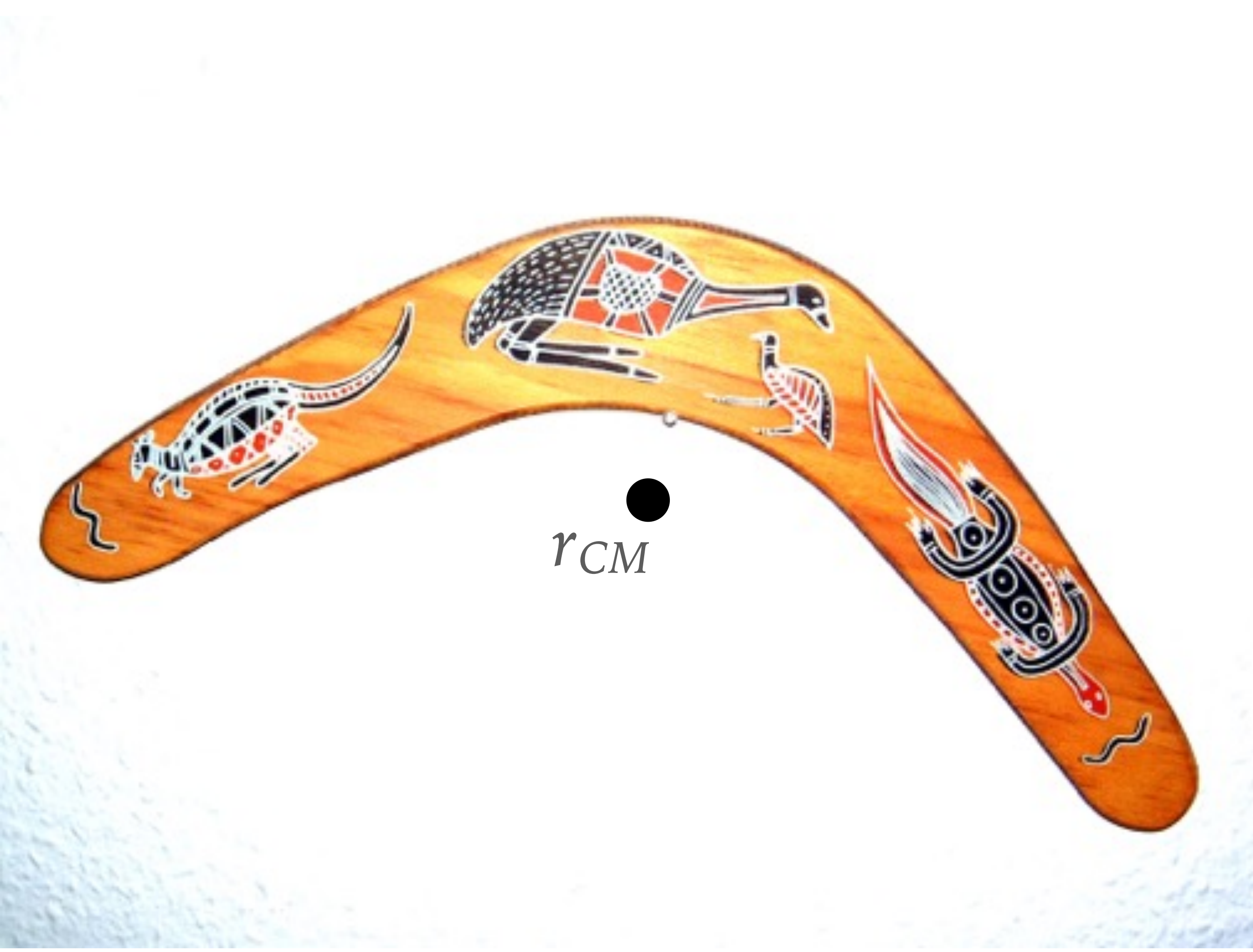
F above CM



F below CM



[*http://www.acs.psu.edu/drussell/Demos/COM/com-a.html*](http://www.acs.psu.edu/drussell/Demos/COM/com-a.html)



THINGS TO NOTE

- For objects and systems in two or three dimensions, need to specify not just $x_{\rm CM}$ but y and z also
- The center of mass does *not* need to be inside the object in question
  - CM found experimentally

http://imgur.com/LgWwoQh

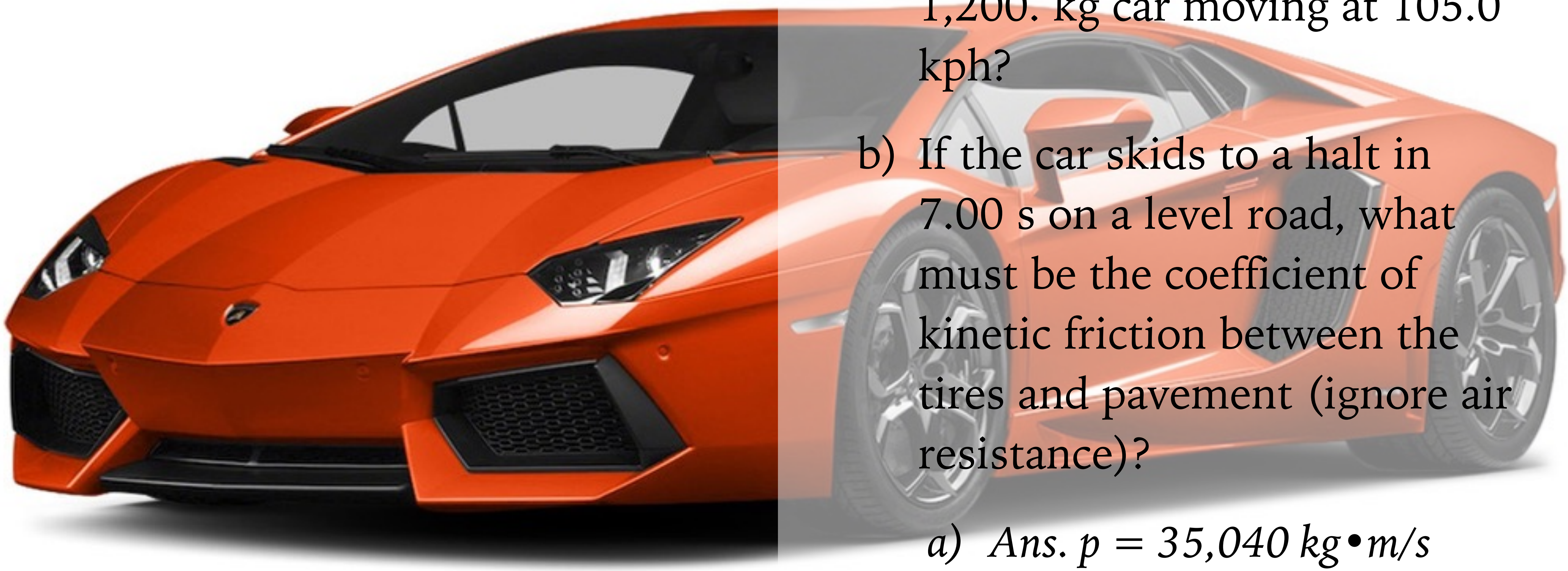
CENTER OF GRAVITY

- Similar concept to center of mass
- Center of Gravity (CG) is the point at which the force of gravity can be considered to act
  - at least for the purposes of determining translational motion
- For practical purposes CM and CG are generally the same point
  - When would they not be?

RECAP

- Momentum, p, is inertia in motion
  - $\mathbf{p} = m\mathbf{v}$
- Newton's 2nd Law can be rewritten in terms of momentum
  - $F_{net} = \Delta \mathbf{p} / \Delta t$
  - i.e. the rate of change of momentum is equal to the net applied force

EXAMPLE 12



a) What is the momentum of a 1,200. kg car moving at 105.0 kph?

b) If the car skids to a halt in 7.00 s on a level road, what must be the coefficient of kinetic friction between the tires and pavement (ignore air resistance)?

a) Ans. $p = 35{,}040\ kg\bullet m/s$

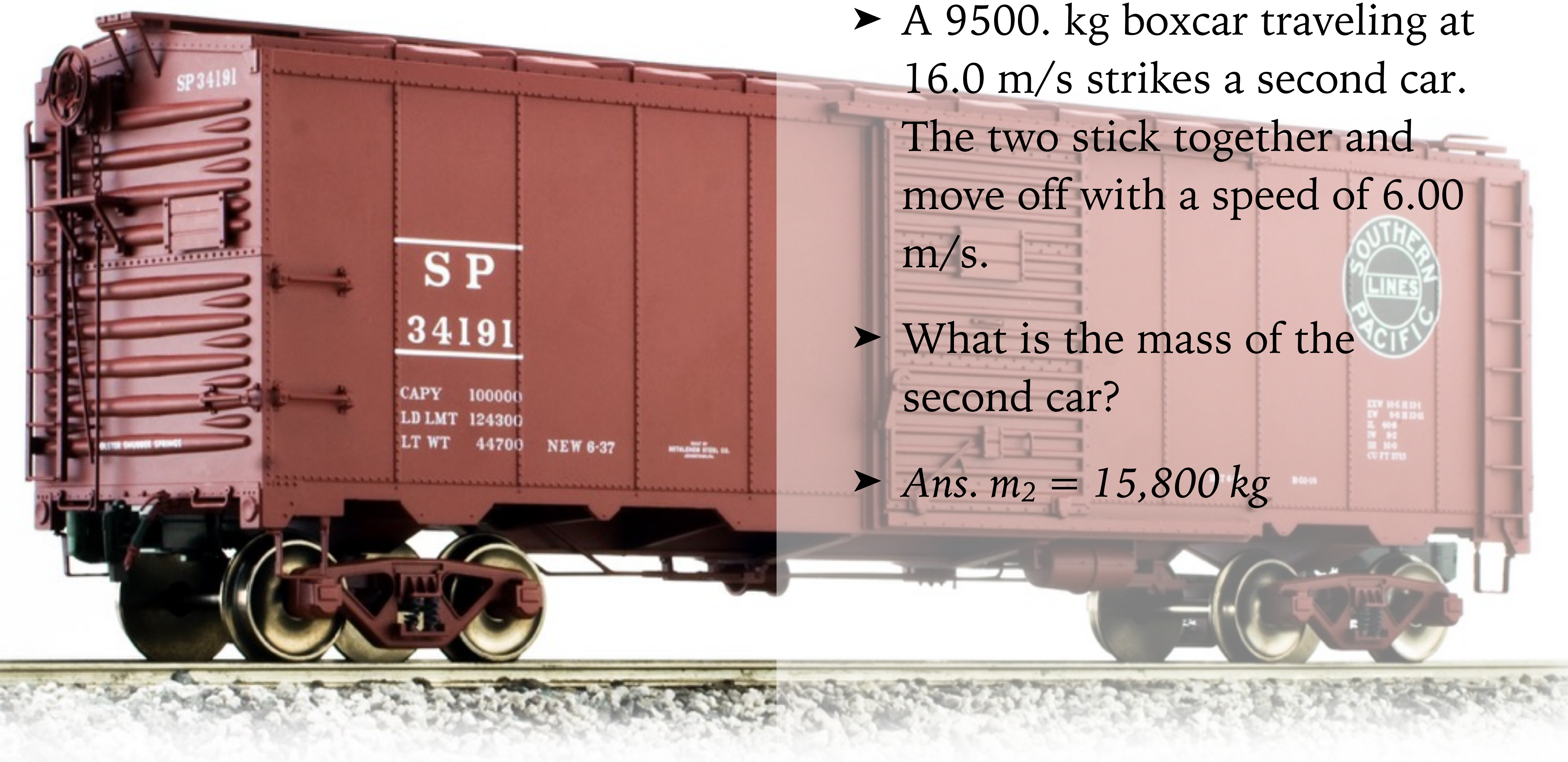
b) Ans. $\mu_k = 0.425$

RECAP

- The law of conservation of momentum states that the total momentum of an isolated system remains constant
- An isolated system is one where there is no net external force
- Really helpful when dealing with collisions

EXAMPLE 13

- A 9500. kg boxcar traveling at 16.0 m/s strikes a second car. The two stick together and move off with a speed of 6.00 m/s.
- What is the mass of the second car?
- *Ans.* $m_2 = 15{,}800\ kg$



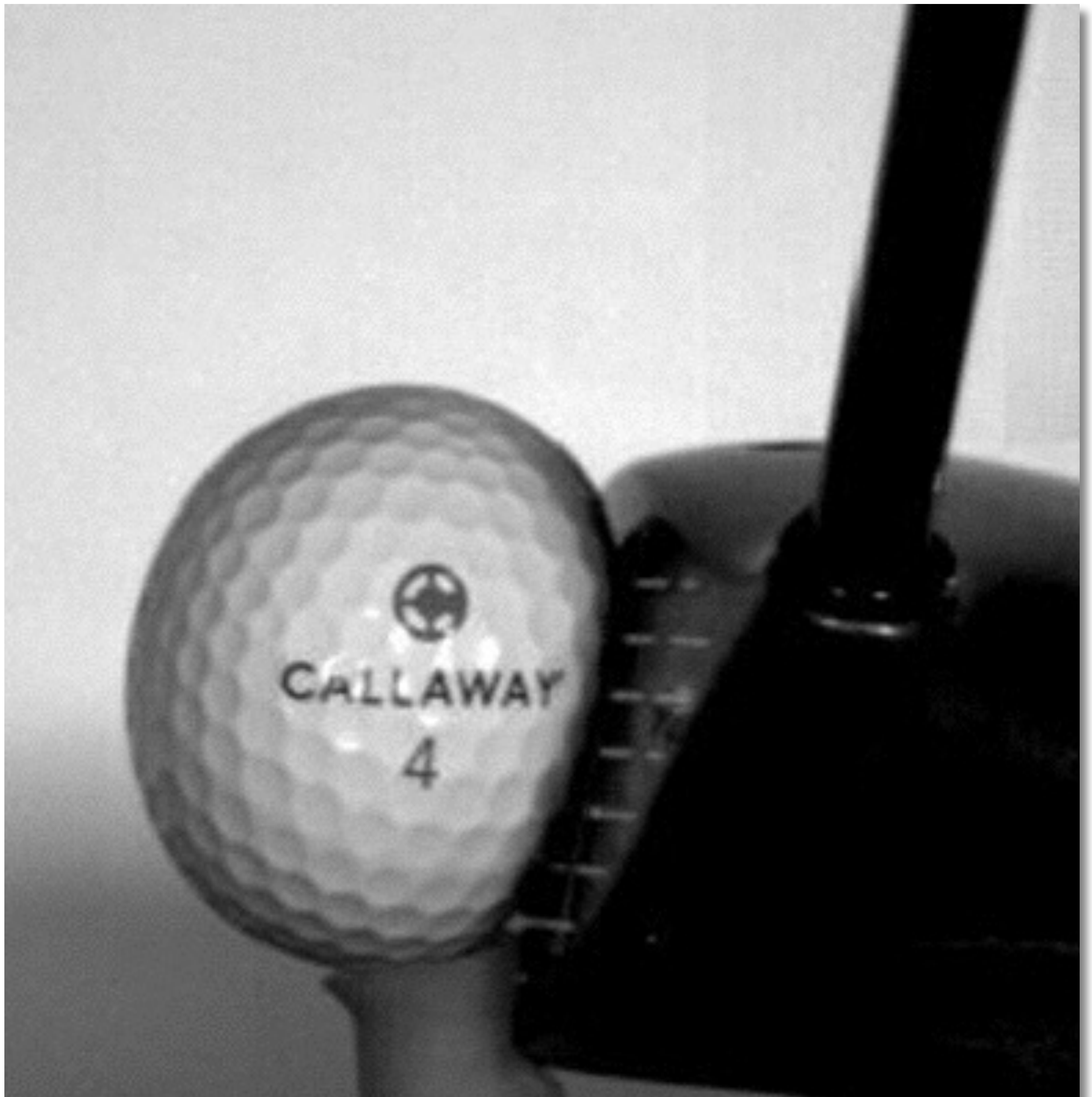
RECAP

- Two types of collisions: elastic and inelastic
- A collision is elastic if *kinetic* energy is also conserved
  - $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1' + m_2\mathbf{v}_2'$
  - $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$
- A perfectly inelastic collision is one in which the colliding bodies stick together after the collision
  - $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}'$

RECAP

- The impulse of a force on a body is defined as $\mathbf{F}\Delta t$
  - **F** is the average force acting during the time Δt
- The impulse is equal to the change in momentum of the body
- $\mathbf{J} = \mathbf{F}_{net}\Delta t = \Delta \mathbf{p}$

EXAMPLE 15

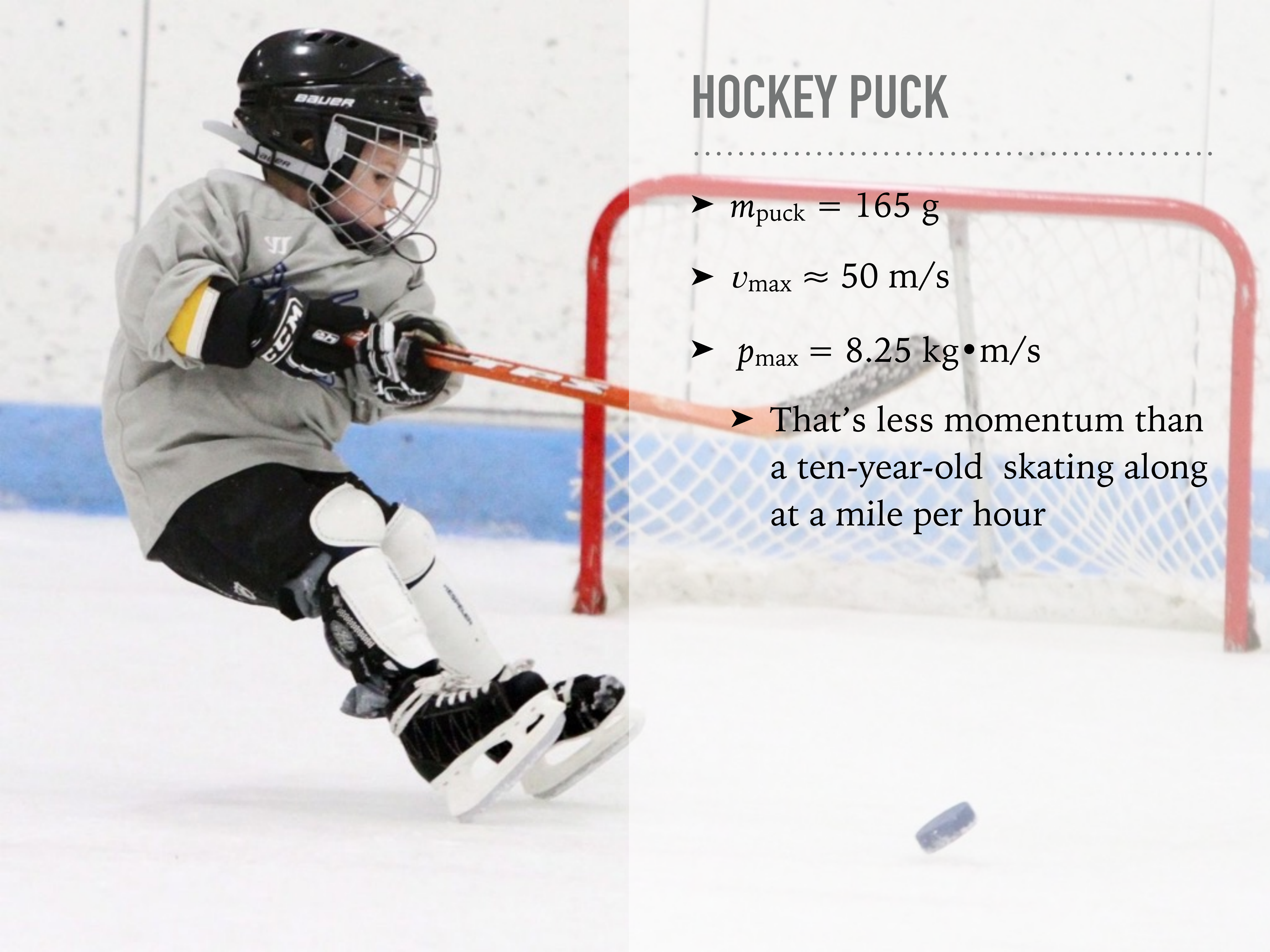


- A golf ball of mass 0.045 kg is hit off the tee at a speed of 45 m/s. The golf club was in contact with the ball for 5.0×10^{-3} s.
- Find (*a*) the impulse imparted to the golf ball, and (*b*) the average force exerted on the ball by the golf club
  - *Ans.* $J = 2.0\ N \cdot s$
  - $F_{avg} = 405\ N$

RECAP

- The center of mass (CM) of an object (or group of objects) is that point at which the net force can be considered to act for purposes of determining the translational motion of the body as a whole
- The complete motion of a body can be described as the translational motion of its center of mass plus rotation (or other internal motion) about its center of mass

How hard would a puck have to be shot to be able to knock the goalie himself backward into the net?



HOCKEY PUCK

- $m_{\text{puck}} = 165$ g
- $v_{\text{max}} \approx 50$ m/s
- $p_{\text{max}} = 8.25$ kg•m/s
  - That's less momentum than a ten-year-old skating along at a mile per hour

HOCKEY PUCK

- $m_{\text{goalie}} = 96$ kg
- $\mu_s = .84$
  - $F_{\text{fr}} = 790$ N
- $t = 0.5$ s
- $J = F_{\text{fr}}\, t = \Delta p = m_{\text{puck}} \Delta v$
- $(790\text{ N})(0.5\text{ s}) = (0.165\text{ kg}) \bullet v$
- $v = 2390\text{ m/s} \approx \text{Mach } 7$

HOCKEY PUCK

- Other problems:
  - The air ahead of the puck would be compressed and heated very rapidly
  - The surface of the puck would start to melt and char
  - Upon impact, the puck would burst apart with the power of a large firecracker or small stick of dynamite
- Imagine throwing the ripe tomato - as hard as you can - at a cake (that's about what would happen)