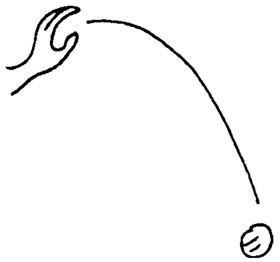
2-2 Projectile Motion

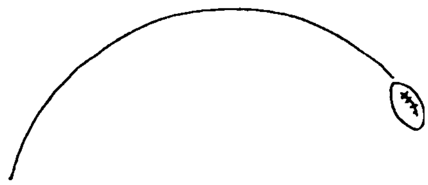
Vocabulary

Projectile: An object that moves through space acted upon only by the earth's gravity.

A projectile may start at a given height and move toward the ground in an arc. For example, picture the path a rock makes when it is tossed straight out from a cliff.



A projectile may also start at a given level and then move upward and downward again as does a football that has been thrown.



Regardless of its path, a projectile will always follow these rules:

1. Projectiles always maintain a constant horizontal velocity (neglecting air resistance).

2. Projectiles always experience a constant vertical acceleration of 10.0 m/s^2 downward (neglecting air resistance).

3. Horizontal and vertical motion are completely independent of each other. Therefore, the velocity of a projectile can be separated into horizontal and vertical components.

4. For a projectile beginning and ending at the same height, the time it takes to rise to its highest point equals the time it takes to fall from the highest point back to the original position.

5. Objects dropped from a moving vehicle have the same velocity as the moving vehicle.

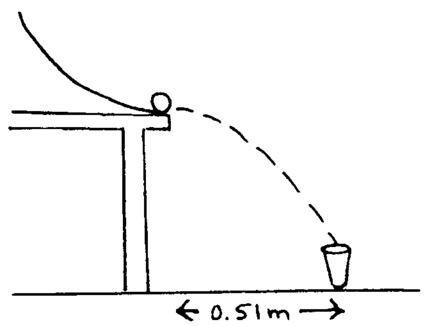
In order to solve projectile exercises, you *must* consider horizontal and vertical motion separately. All of the equations for linear motion in Chapter 1 can be used for projectile motion as well. You don't need to learn any new equations!

To simplify calculations, the term for initial vertical velocity, v_{yo}, will be left out of all equations in which an object is projected horizontally. For example, $\Delta d_y = v_{yo}\Delta t + \left(\frac{1}{2}\right)g\Delta t^2$ will be written as $\Delta d_y = \left(\frac{1}{2}\right)g\Delta t^2$.

Solved Examples

Example 6: In her physics lab, Melanie rolls a 10-g marble down a ramp and off the table with a horizontal velocity of 1.2 m/s. The marble falls in a cup placed 0.51 m from the table's edge. How high is the table?

Solution: The first thing you should notice about projectile exercises is that you do not need to consider the mass of the object projected. Remember, if you ignore air resistance, all bodies fall at exactly the same rate regardless of their mass.



Before you can find the height of the table, you must first determine how long the marble is in the air. The horizontal distance traveled equals the constant horizontal velocity times the travel time.

Given: $\Delta d_x = 0.51\text{ m}$
$v_x = 1.2\text{ m/s}$

Unknown: $\Delta t = ?$
Original equation: $v_x = \dfrac{\Delta d_x}{\Delta t}$

Solve: $\Delta t = \dfrac{\Delta d_x}{v_x} = \dfrac{0.51\text{ m}}{1.2\text{ m/s}} = 0.43\text{ s}$

Now that you know the time the marble takes to fall, you can find the vertical distance it traveled.

Given: $g = 10.0\text{ m/s}^2$
$\Delta t = 0.43\text{ s}$

Unknown: $\Delta d_y = ?$
Original equation: $\Delta d_y = \left(\dfrac{1}{2}\right) g \Delta t^2$

Solve: $\Delta d_y = \left(\dfrac{1}{2}\right)(10.0\text{ m/s}^2)(0.43\text{ s})^2 = \mathbf{0.92\text{ m}}$

Example 7: Bert is standing on a ladder picking apples in his grandfather's orchard. As he pulls each apple off the tree, he tosses it into a basket that sits on the ground 3.0 m below at a horizontal distance of 2.0 m from Bert. How fast must Bert throw the apples (horizontally) in order for them to land in the basket?

Solution: Before you can find the horizontal component of the velocity, you must first find the time that the apple is in the air.

Given: $\Delta d_y = 3.0\text{ m}$
$g = 10.0\text{ m/s}^2$

Unknown: $\Delta t = ?$
Original equation: $\Delta d_y = \left(\dfrac{1}{2}\right) g \Delta t^2$

Solve: $t = \sqrt{\dfrac{2\Delta d_y}{g}} = \sqrt{\dfrac{2(3.0\text{ m})}{10.0\text{ m/s}^2}} = \mathbf{0.77\text{ s}}$

Now that you know the time, you can use it to find the horizontal component of the velocity.

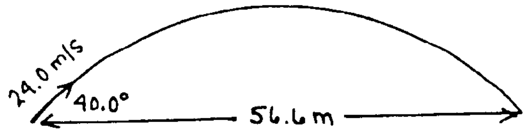
Given: $\Delta d_x = 2.0\text{ m}$
$\Delta t = 0.77\text{ s}$

Unknown: $v_x = ?$
Original equation: $\Delta d_x = v_x \Delta t$

Solve: $v_x = \dfrac{\Delta d_x}{\Delta t} = \dfrac{2.0\text{ m}}{0.77\text{ s}} = \mathbf{2.6\text{ m/s}}$

Example 8: Emanuel Zacchini, the famous human cannonball of the Ringling Bros. and Barnum & Bailey Circus, was fired out of a cannon with a speed of 24.0 m/s at an angle of 40.0° to the horizontal. If he landed in a net 56.6 m away at the same height from which he was fired, how long was Zacchini in the air?

Solution: Because Zacchini was in the air for the same amount of time vertically that he was horizontally, you can find his horizontal time and this will be the answer. First, you need the horizontal velocity component.



$$\cos\theta = \frac{v_x}{v} \qquad v_x = v\cos\theta = (24.0\text{ m/s})\cos 40.0° = 18.4\text{ m/s}$$

Now you have the horizontal velocity component and the horizontal displacement, so you can find the time.

Given: $v_x = 18.4\text{ m/s}$
$\Delta d_x = 56.6\text{ m}$

Unknown: $\Delta t = ?$
Original equation: $\Delta d_x = v_x \Delta t$

Solve: $\Delta t = \dfrac{\Delta d_x}{v_x} = \dfrac{56.6\text{ m}}{18.4\text{ m/s}} = \mathbf{3.08\ s}$

Example 9: On May 20, 1999, 37-year old Robbie Knievel, son of famed daredevil Evel Knievel, successfully jumped 69.5 m over a Grand Canyon gorge. Assuming that he started and landed at the same level and was airborne for 3.66 s, what height from his starting point did this daredevil achieve?

Solution: Because 3.66 s is the time for the entire travel through the air, Robbie spent half of this time reaching the height of the jump. The motorcycle took 1.83 s to go up, and another 1.83 s to come down. To find the height the motorcycle achieved, look only at its downward motion as measured from the highest point.

Given: $\Delta t = 1.83\text{ s}$
$g = 10.0\text{ m/s}^2$

Unknown: $\Delta d_y = ?$
Original equation: $\Delta d_y = \left(\frac{1}{2}\right) g \Delta t^2$

Solve: $\Delta d_y = \left(\frac{1}{2}\right) g \Delta t^2 = \left(\frac{1}{2}\right)(10.0\text{ m/s}^2)(1.83\text{ s})^2 = \mathbf{16.7\ m}$

Practice Exercises

Exercise 10: Billy-Joe stands on the Talahatchee Bridge kicking stones into the water below. a) If Billy-Joe kicks a stone with a horizontal velocity of 3.50 m/s, and it lands in the water a horizontal distance of 5.40 m from where Billy-Joe is standing, what is the height of the bridge? b) If the stone had been kicked harder, how would this affect the time it would take to fall?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 11: The movie "The Gods Must Be Crazy" begins with a pilot dropping a bottle out of an airplane. It is recovered by a surprised native below, who thinks it is a message from the gods. If the plane from which the bottle was dropped was flying at an altitude of 500. m, and the bottle lands 400. m horizontally from the initial dropping point, how fast was the plane flying when the bottle was released?

Answer: ____________________

Exercise 12: Tad drops a cherry pit out the car window 1.0 m above the ground while traveling down the road at 18 m/s. a) How far, horizontally, from the initial dropping point will the pit hit the ground? b) Draw a picture of the situation. c) If the car continues to travel at the same speed, where will the car be in relation to the pit when it lands?

Answer: **a.** ____________________

Answer: **c.** ____________________

Exercise 13: Ferdinand the frog is hopping from lily pad to lily pad in search of a good fly for lunch. If the lily pads are spaced 2.4 m apart, and Ferdinand jumps with a speed of 5.0 m/s, taking 0.60 s to go from lily pad to lily pad, at what angle must Ferdinand make each of his jumps?

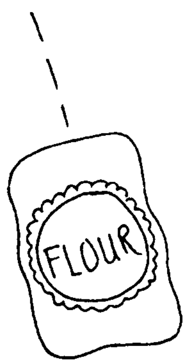
Answer: ____________________

Exercise 14: At her wedding, Jennifer lines up all the single females in a straight line away from her in preparation for the tossing of the bridal bouquet. She stands Kelly at 1.0 m, Kendra at 1.5 m, Mary at 2.0 m, Kristen at 2.5 m, and Lauren at 3.0 m. Jennifer turns around and tosses the bouquet behind her with a speed of 3.9 m/s at an angle of 50.0° to the horizontal, and it is caught at the same height 0.60 s later. a) Who catches the bridal bouquet? b) Who might have caught it if she had thrown it more slowly?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 15: At a meeting of physics teachers in Montana, the teachers were asked to calculate where a flour sack would land if dropped from a moving airplane. The plane would be moving horizontally at a constant speed of 60.0 m/s at an altitude of 300. m. a) If one of the physics teachers neglected air resistance while making his calculation, how far horizontally from the dropping point would he predict the landing? b) Draw a sketch that shows the path the flour sack would take as it falls to the ground (from the perspective of an observer on the ground and off to the side.)



Answer: **a.** ____________________

Exercise 16: Jack be nimble, Jack be quick, Jack jumped over the candlestick with a velocity of 5.0 m/s at an angle of 30.0° to the horizontal. Did Jack burn his feet on the 0.25-m-high candle?

Answer: ____________________

11. 40.0 m/s
13. 37° to horizontal
15. 465 m