11 Simple Harmonic Motion

11-1 Springs

Vocabulary **Period:** The time it takes for a vibrating object to repeat its motion.

Vocabulary **Frequency:** The number of vibrations made per unit time.

Period and frequency are the reciprocals of each other. In other words,

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

Since period is a measure of time, its SI unit is the **second,** while the unit for frequency is the reciprocal of this, or **1/second.** Another way of writing 1/s is with the unit **hertz (Hz).**

You may recognize this as being similar to the explanation of period and frequency in Chapter 6 on circular motion.

Hooke's Law

Whenever a spring is stretched from its equilibrium position and released, it will move back and forth on either side of the equilibrium position. The force that pulls it back and attempts to restore the spring to equilibrium is called the **restoring force.** Its magnitude can be written as

restoring force = (**force constant**)(**displacement from equilibrium**)
or $F = kx$

This relationship is known as **Hooke's law.** The force constant is a measure of the stiffness of the spring. The SI unit for the force constant is the **newton per meter (N/m).**

Keep in mind that this is the force required to restore the spring back to its original position. The force that acts to move the spring *away* from the equilibrium position is equal in magnitude to the restoring force, but opposite in direction.

Simple harmonic motion is motion that occurs when the restoring force acting on an object is proportional to the object's displacement from its rest position. Objects at the end of springs move in simple harmonic motion when they are displaced from their rest position and bounce up and down on the spring, or oscillate.

Period of a Mass on a Spring in Simple Harmonic Motion

The only two things that affect the period of an object hanging on a bouncing spring are the object's mass and the force constant of the spring on which the object is oscillating.

$$\textbf{Period} = \boldsymbol{2\pi}\sqrt{\frac{\textbf{mass}}{\textbf{force constant}}} \qquad \text{or} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

To prove that this equation does indeed give the period in seconds, simplify the units for $\sqrt{m/k}$ by writing

$$\sqrt{\frac{\text{kg}}{\text{N/m}}} = \sqrt{\frac{\text{kg}}{\frac{\text{kg}\cdot\text{m/s}^2}{\text{m}}}} = \sqrt{\text{s}^2} = \text{s}$$

Solved Examples

Example 1: A hummingbird beats its wings up and down with a frequency of 80.0 Hz. What is the period of the hummingbird's flaps?

Given: $f = 80.0$ Hz

Unknown: $T = ?$
Original equation: $T = \dfrac{1}{f}$

Solve: $T = \dfrac{1}{f} = \dfrac{1}{80.0\text{ Hz}} = \mathbf{0.0125\ s}$

Example 2: In anticipation of her first game, Alesia pulls back the handle of a pinball machine a distance of 5.0 cm. The force constant of the spring in the handle is 200 N/m. How much force must Alesia exert?

Solution: First, convert cm to m. 5.0 cm = 0.050 m

Given: $k = 200$ N/m
$x = 0.050$ m

Unknown: $F = ?$
Original equation: $F = kx$

Solve: $F = kx = (200\text{ N/m})(0.050\text{ m}) = \mathbf{10\ N}$

Example 3: As Bianca stands on a bathroom scale, whose force constant is 220 N/m, the needle on the scale vibrates from side to side. a) If Bianca has a mass of 180 kg, what is the period of vibration of the needle as it comes to rest? b) If Bianca goes on a diet, how will this change the period of vibration?

a. *Given:* $m = 180$ kg
$k = 220$ N/m

Unknown: $T = ?$
Original equation: $T = 2\pi\sqrt{\dfrac{m}{k}}$

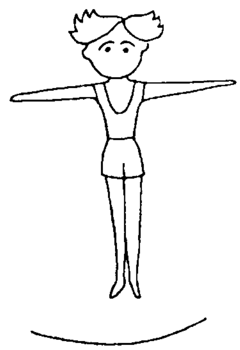
Solve: $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{180\ \text{kg}}{220\ \text{N/m}}} = \mathbf{5.7\ s}$

In other words, this is the amount of time for one complete oscillation.

b. Because the mass will be smaller, the period of vibration will be smaller. In other words, it will take less time for the needle on the scale to bounce from side to side as it comes to rest.

Practice Exercises

Exercise 1: Terry jumps up and down on a trampoline with a frequency of 1.5 Hz. What is the period of Terry's jumping?



Answer: ____________________

Exercise 2: Gary Stewart of Reading, Ohio set a pogo stick record in 1990 by jumping 177 737 times. a) If the pogo stick he used had a force constant of 6000. N/m and was compressed 0.12 m on each jump, what force must Gary have exerted on the pogo stick upon each jump? b) What force would be exerted back up on Gary each time he went up?

Answer: **a.** ____________________

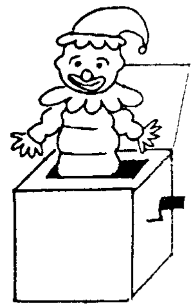
Answer: **b.** ____________________

Exercise 3: At the post office, Cliff, a postal worker, places a 0.60-kg package on a scale, compressing the scale by 0.03 m. a) What is the force constant of the spring in the postal scale? b) What happens to the force constant if Cliff weighs a heavier package?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 4: A jack-in-the-box lid will pop open when a crank is turned on the outside of the box. If Jack pushes against the inside of the box with a force of 3.00 N when the lid is closed, and the spring is compressed 10.0 cm from equilibrium, what is the force constant of the spring?



Answer: ____________________

Exercise 5: Sam, a butcher, puts 3.0 kg of chopped beef on the 1.0-kg pan of his scale, which has a spring whose force constant is 400. N/m. What is the period of vibration of the pan as it comes to rest? b) If Sam adds more beef to the scale, what will this do to the period of vibration?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 6: A toy bobs up and down over Campbell's crib with a period of 1.0 s. The toy hangs from the end of a spring whose force constant is 20.0 N/m. What is the mass of the toy?

Answer: ______________________

Chapter 11

1. 0.67 s
3. a) 200 N/m
5. a) 0.63 s