11-2 Pendulums

The period of a pendulum depends only upon the pendulum's length (if the angle of swing is not too large). A long pendulum has a longer period than a short pendulum. The relationship between period and length can be shown with the following equation.

$$\textbf{Period} = 2\pi\sqrt{\frac{\textbf{length}}{\textbf{acceleration due to gravity}}} \qquad \text{or} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

It should be noted that this equation works only for a pendulum whose mass is considerably larger than the mass of the string from which it swings. To simplify calculations, in the following exercises you will be working with pendulums swinging from strings of negligible mass.

Solved Examples

Example 4: A tall, thin tree sways back and forth in the breeze with a frequency of 2 Hz. What is the period of the tree?

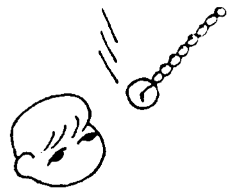
Given: $f = 2\text{ Hz}$

Unknown: $T = ?$

Original equation: $T = \dfrac{1}{f}$

Solve: $T = \dfrac{1}{f} = \dfrac{1}{2\text{ Hz}} = \mathbf{0.5\ s}$

Example 5: World-reknowned hypnotist Paulbar the Great swings his watch from a 20.0-cm chain in front of a subject's eyes. What is the period of swing of the watch?



Solution: First, convert cm to m. 20.0 cm = 0.20 m

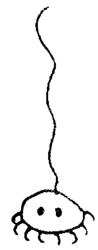
Given: $L = 0.20\text{ m}$
$g = 10.0\text{ m/s}^2$

Unknown: $T = ?$
Original equation: $T = 2\pi\sqrt{\dfrac{L}{g}}$

Solve: $T = 2\pi\sqrt{\dfrac{L}{g}} = 2\pi\sqrt{\dfrac{0.20\text{ m}}{10.0\text{ m/s}^2}} = \mathbf{0.89\text{ s}}$

Therefore, it takes 0.89 s for the watch to swing in one direction and back again, through one full cycle.

Example 6: A spider swings in the breeze from a silk thread with a period of 0.6 s. How long is the spider's strand of silk?



Solution: The answer is determined using the pendulum equation, but now it must be set up in terms of the unknown, L. First, square all of the terms to get rid of the radical. The equation becomes

$T^2 = 4\pi^2\dfrac{L}{g}$. Then rearrange the equation as shown.

Given: $T = 0.60\text{ s}$
$g = 10.0\text{ m/s}^2$

Unknown: $L = ?$
Original equation: $T = 2\pi\sqrt{\dfrac{L}{g}}$

Solve: $L = \dfrac{gT^2}{4\pi^2} = \dfrac{(10.0\text{ m/s}^2)(0.6\text{ s})^2}{4\pi^2} = \mathbf{0.09\text{ m}}$

Practice Exercises

Exercise 7: A metronome is a device used by many musicians to get the desired rhythm for a musical piece. If a metronome is clicking back and forth with a frequency of 0.5 Hz, what is the period of the metronome?

Answer: ______________________

Exercise 8: Many amusement parks feature a ride in which a giant ship swings back and forth. If the period of the ship is 8.00 s, what is the frequency of the swinging ship?

Answer: ____________________

Exercise 9: Tegan, a trapeze artist, swings from a 2.5-m-long trapeze, high above the three-ring circus. a) What is Tegan's period of swing? b) Would Tegan's period of swing change if she were more massive? If so, how?

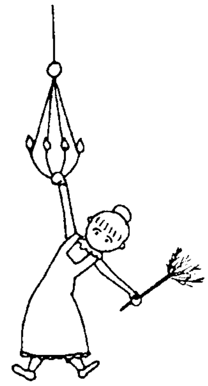
Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 10: Danielle is pushing her twin Daniel on a swing that hangs from a tree branch by 2.0-m-long ropes. With what frequency will Danielle have to push Daniel as he swings?

Answer: ____________________

Exercise 11: Marla, a maid, is standing on the Vanderbilt's dining room table dusting the chandelier. While Marla is reaching up, she slips and grabs hold of the chandelier to catch her balance. When she lets go, the chandelier begins to swing with a period of 1.6 s. How long is the cable connecting the chandelier to the ceiling?



Answer: ____________________

Exercise 12: You have been commissioned by NASA to travel to Jupiter's innermost Galilean satellite, Io, to learn more about this volcanic moon. As you board the spacecraft, you are handed a rock tied to a 10.0-cm string, and a stopwatch, and are asked to derive an experiment that would allow you to determine the acceleration due to gravity on Io. You must use both pieces of equipment and nothing more. a) Describe how you would calculate Io's gravitational acceleration. b) If the pendulum swings with a period of 1.48 s, what is the gravitational acceleration on Io?

Answer: **a.** ____________________

Answer: **b.** ____________________

5. a) 0.63 s
7. 2 s
9. a) 3.1 s
11. 0.65 m