16.4 Series and Parallel Circuits

When multiple resistors are used in a circuit, the total resistance in the circuit must be found before finding the current. Resistors can be combined in a circuit in series or in parallel.

Resistors in Series

When connected in series, the total resistance, R_T, is equal to

$$R_T = R_1 + R_2 + R_3 + \ldots$$

In series, the total resistance is always *larger* than any individual resistance.

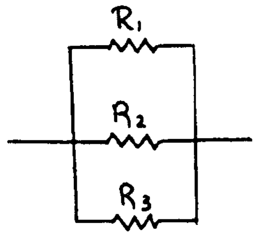
Current in series resistors: In series circuits, charge has only one path through which to flow. Therefore, the current passing through each resistor in series is the same.

Potential difference across series resistors: As charge passes through each of the resistors, it loses some energy. This means that there will be a potential difference across each resistor. The sum of all the potential differences equals the potential difference across the battery, assuming negligible resistance in the connecting wires.

Resistors in Parallel

When connected in parallel, the total resistance, R_T, is equal to

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$



Don't forget! After finding a common denominator and determining the sum of these fractions, flip over the answer to determine R_T.

In parallel circuits, the total resistance is always *smaller* than any individual resistance.

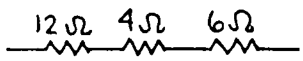
Current in parallel resistors: In parallel circuits, there is more than one possible path and current divides itself according to the resistance of each path. Since current will take the "path of least resistance," the smallest resistor will allow the most current through, while the largest resistor will allow the least current through. The sum of the currents in each parallel resistor equals the original current entering the branches.

Potential difference in parallel resistors: The potential difference across each of the resistors in a parallel combination is the same. If there are no other resistors in the circuit, it is equal to the potential difference across the battery, assuming negligible resistance in the connecting wires.

Solved Examples

Example 8: Find the total resistance of the three resistors connected in series.

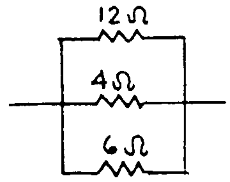
Solve: $R_T = R_1 + R_2 + R_3 = 12\ \Omega + 4\ \Omega + 6\ \Omega = \mathbf{22\ \Omega}$



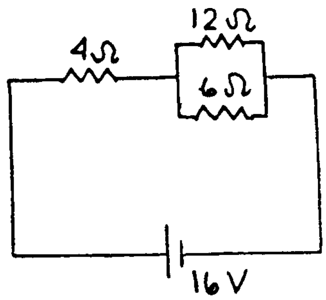
Example 9: Find the total resistance of the same three resistors now connected in parallel.

Solve: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{12\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{6\ \Omega}$

$\frac{1}{R_T} = \frac{1}{12\ \Omega} + \frac{3}{12\ \Omega} + \frac{2}{12\ \Omega} = \frac{6}{12\ \Omega} = \frac{1}{2\ \Omega}$ $R_T = \mathbf{2\ \Omega}$



Example 10: Find the total current passing through the circuit.



This circuit contains resistors in parallel that are then combined with a resistor in series. Always begin solving such a resistor combination by working from the inside out. In other words, first determine the equivalent resistance of the two resistors in parallel before combining this total resistance with the one in series.

Look first at the parallel combination.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12\ \Omega} + \frac{1}{6\ \Omega} = \frac{1}{12\ \Omega} + \frac{2}{12\ \Omega} = \frac{3}{12\ \Omega} = \frac{1}{4\ \Omega}$$

$R_T = \mathbf{4}\ \Omega$

Now, combine this equivalent resistance with the resistor in series.

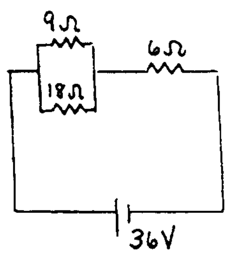
$$R_T = R_1 + R_2 = 4\ \Omega + 4\ \Omega = \mathbf{8}\ \Omega$$

To find the current flowing through the circuit, use this total resistance in combination with the potential difference from the battery.

Given: $V = 16$ V
$R = 8\ \Omega$

Unknown: $I = ?$
Original equation: $V = IR$

Solve: $I = \frac{V}{R} = \frac{16\ \text{V}}{8\ \Omega} = \mathbf{2\ A}$



Example 11: Find the current in the 9-Ω resistor.

For the parallel branch

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{9\ \Omega} + \frac{1}{18\ \Omega} = \frac{2}{18\ \Omega} + \frac{1}{18\ \Omega} = \frac{3}{18\ \Omega} = \frac{1}{6\ \Omega}$$

$R_T = \mathbf{6}\ \Omega$

Combining with the series resistor

$$R_T = R_1 + R_2 = 6\ \Omega + 6\ \Omega = \mathbf{12}\ \Omega$$

Given: $V = 36\text{ V}$ | *Unknown:* $I = ?$
$R = 12\ \Omega$ | *Original equation:* $V = IR$

Solve: $I = \dfrac{V}{R} = \dfrac{36\text{ V}}{12\ \Omega} = \mathbf{3\ A}$

This 3 A is the current through the entire circuit. Use this current to find the potential difference across the parallel combination. Remember, the potential difference across resistors wired in parallel is the same regardless of which path is taken. Because the resistors in parallel have a combined resistance of 6 Ω, you find the potential difference across the parallel branch as follows.

Given: $R = 6\ \Omega$ | *Unknown:* $V = ?$
$I = 3\text{ A}$ | *Original equation:* $V = IR$

Solve: $V = IR = (3\text{ A})(6\ \Omega) = \mathbf{18\ V}$

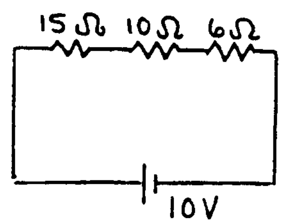
Therefore, the potential difference across both the top and the bottom branches is 18 V. Now use this 18-V drop to determine the current in the 9-Ω resistor.

Given: $V = 18\text{ V}$ | *Unknown:* $I = ?$
$R = 9\ \Omega$ | *Original equation:* $V = IR$

Solve: $I = \dfrac{V}{R} = \dfrac{18\text{ V}}{9\ \Omega} = \mathbf{2\ A}$

Practice Exercises

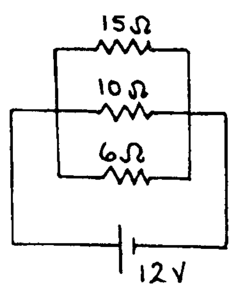
Exercise 16: Using the diagram, a) find the total resistance in the circuit. b) Find the total current through the circuit.



Answer: **a.** ____________________

Answer: **b.** ____________________

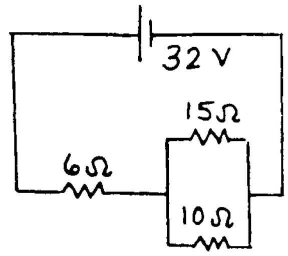
Exercise 17: Using the diagram, a) find the total resistance in the circuit. b) Find the total current through the circuit.



Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 18: Using the diagram, a) find the total resistance in the circuit. b) Find the total current through the circuit.



Answer: **a.** ____________________

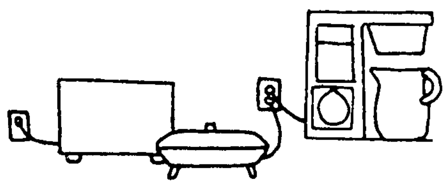
Answer: **b.** ____________________

Exercise 19: Old-fashioned holiday lights were connected in series across a 120-V household line. a) If a string of these lights consists of 12 bulbs, what is the potential difference across each bulb? b) If the bulbs were connected in parallel, what would be the potential difference across each bulb?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 20: Before going to work each morning, Gene runs his 18-Ω toaster, 11-Ω electric frying pan, and 14-Ω electric coffee maker, all at the same time. The three are connected in parallel across a 120-V line. a) What is the current through each appliance? b) If a household circuit could carry a maximum current of 15 A, would Gene be able to run all of these appliances at the same time?



Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 21: Timmy is playing with a new electronics kit he has received for his birthday. He takes out four resistors with resistances of 15 Ω, 20 Ω, 20 Ω, and 30 Ω. a) How would Timmy have to wire the resistors so that they would allow the maximum amount of current to be drawn? Calculate the total resistance in this circuit. b) How must he wire the resistors so that they draw a minimum amount of current? Calculate the total resistance in this circuit.

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 22: Farmer Crockett is preparing tomato seedlings for his spring planting by growing the small plants over five 46-Ω strip heaters wired in parallel. a) How much current does each heater draw from a 120-V line? b) How much current do they draw all together?

Answer: **a.** ____________________

Answer: **b.** ____________________

17. a) 3 Ω
    b) 4 A
19. Series: 10. V
    Parallel: 120 V
21. a) 5 Ω
    b) 85 Ω