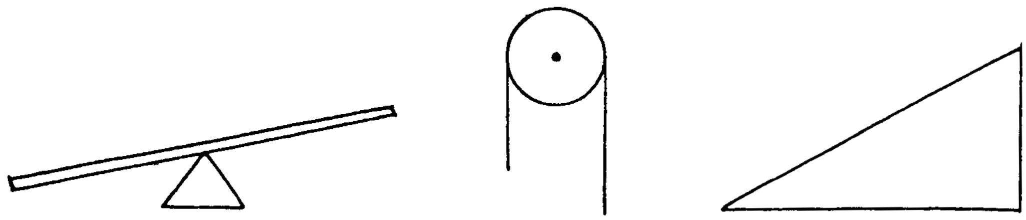
5-3 Machines and Efficiency

Vocabulary **Machine:** A device that helps do work by changing the magnitude or direction of the applied force.

Three common machines are the **lever, pulley,** and **incline.**



In an ideal situation, where frictional forces are negligible, work input equals work output.

$$F_{in}\Delta d_{in} = F_{out}\Delta d_{out}$$

However, situations are never ideal. The **actual mechanical advantage,** or **AMA,** of the machine is a ratio of the magnitude of the force out (resistance) to the magnitude of the force in (effort).

$$\textbf{actual mechanical advantage} = \frac{\textbf{force out (resistance)}}{\textbf{force in (effort)}} \quad \text{or} \quad \text{AMA} = \frac{F_{out}}{F_{in}}$$

On the other hand, the theoretical or **ideal mechanical advantage, IMA,** is based only on the geometry of the system and does not take frictional effects into account.

$$\textbf{ideal mechanical advantage} = \frac{\textbf{distance in (effort distance)}}{\textbf{distance out (resistance distance)}}$$

$$\text{or} \quad \text{IMA} = \frac{\Delta d_{in}}{\Delta d_{out}}$$

Because no machine is perfect and because you will always get out less work than you put in, you need to consider the efficiency of the machine that you are using. The more efficient the machine, the greater work output you will get for your work input. The efficiency will always be less than 100%.

Vocabulary **Efficiency:** The ratio of the work output to the work input.

$$\textbf{efficiency} = \frac{\textbf{work output}}{\textbf{work input}} = \frac{F_{out}\Delta d_{out}}{F_{in}\Delta d_{in}} = \frac{\text{AMA}}{\text{IMA}}$$

Efficiency has no units and is usually expressed as a percent.

Solved Examples

Example 8: A crate of bananas weighing 3000. N is shipped from South America to New York, where it is unloaded by a dock worker who lifts the crate by pulling with a force of 200. N on the rope of a pulley system. What is the actual mechanical advantage of the pulley system?

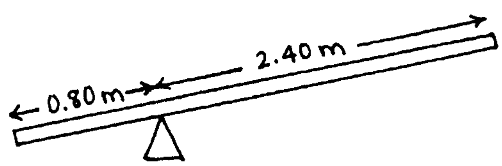
Given: $F_{out} = 3000.\ \text{N}$
$F_{in} = 200.\ \text{N}$

Unknown: AMA = ?
Original equation: $\text{AMA} = \frac{F_{out}}{F_{in}}$

Solve: $\text{AMA} = \frac{F_{out}}{F_{in}} = \frac{3000.\ \text{N}}{200.\ \text{N}} = \mathbf{15.0}$

The pulley exerts 15.0 times more force on the crate than the dock worker exerts to pull the rope. Notice that mechanical advantage has no units.

Example 9: Two clowns, of mass 50.0 kg and 70.0 kg respectively, are in a circus act performing a stunt with a trampoline and a seesaw. The smaller clown stands on the lower end of the seesaw while the larger clown jumps from the trampoline onto the raised side of the seesaw, propelling his friend into the air. a) what is the ideal mechanical advantage of the seesaw? b) If the larger clown exerts a force of 850. N on the seesaw as he jumps, how much force is exerted on the smaller clown?



a. The seesaw acts as a lever with the fulcrum 0.80 m from the left side. The ideal mechanical advantage is found by comparing the two distances.

Given: $\Delta d_{in} = 2.40\ \text{m}$
$\Delta d_{out} = 0.80\ \text{m}$

Unknown: IMA = ?
Original equation: $\text{IMA} = \frac{\Delta d_{in}}{\Delta d_{out}}$

Solve: $\text{IMA} = \frac{\Delta d_{in}}{\Delta d_{out}} = \frac{2.40\ \text{m}}{0.80\ \text{m}} = \mathbf{3.0}$

b. To answer this question, assume that the seesaw is 100% efficient and the work out equals the work in (which is highly unlikely!).

Given: $F_{\text{in}} = 850.\ \text{N}$
$\Delta d_{\text{in}} = 2.40\ \text{m}$
$\Delta d_{\text{out}} = 0.80\ \text{m}$

Unknown: $F_{\text{out}} = ?$
Original equation: $F_{\text{in}}\Delta d_{\text{in}} = F_{\text{out}}\Delta d_{\text{out}}$

Solve: $F_{\text{out}} = \dfrac{F_{\text{in}}\Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \dfrac{(850.\ \text{N})(2.40\ \text{m})}{0.80\ \text{m}} = \mathbf{2550\ N}$

Example 10: A jackscrew with a handle 30.0 cm long is used to lift a car sitting on the jack. The car rises 2.0 cm for every full turn of the handle. What is the ideal mechanical advantage of the jack?

Solution: For a screw, IMA $= \dfrac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \dfrac{2\pi r}{\Delta h}$ where $2\pi r$ is the circumference of the circle through which the handle turns, and height, Δh, refers to the amount the jack (and hence the automobile) is raised.

Given: $r = 30.0\ \text{cm}$
$\Delta h = 2.0\ \text{cm}$

Unknown: IMA = ?
Original equation: IMA $= \dfrac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}}$

Solve: IMA $= \dfrac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \dfrac{2\pi r}{\Delta h} = \dfrac{2\pi(30.0\ \text{cm})}{2.0\ \text{cm}} = \mathbf{94}$

Example 11: Jack and Jill went up the hill to fetch a pail of water. At the well, Jill used a force of 20.0 N to turn a crank handle of radius 0.400 m that rotated an axle of radius 0.100 m, so she could raise a 60.0-N bucket of water. a) What is the ideal mechanical advantage of the wheel? b) What is the actual mechanical advantage of the wheel? c) What is the efficiency of the wheel?

Solution: Since the crank handle and the axle both turn in a circle, $\Delta d_{\text{in}} = 2\pi r_{\text{c}}$ (where r_{c} is the radius of the crank handle) and $\Delta d_{\text{out}} = 2\pi r_{\text{a}}$ (where r_{a} is the radius of the axle).

a. *Given:* $r_{\text{c}} = 0.400\ \text{m}$
$r_{\text{a}} = 0.100\ \text{m}$

Unknown: IMA = ?
Original equation: IMA $= \dfrac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}}$

Solve: IMA $= \dfrac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \dfrac{2\pi r_{\text{c}}}{2\pi r_{\text{a}}} = \dfrac{2\pi(0.400\ \text{m})}{2\pi(0.100\ \text{m})} = \mathbf{4.00}$

b. The force on the bucket of water is F_{out} and the force exerted by Jill is F_{in}.

Given: $F_{\text{out}} = 60.0\ \text{N}$
$F_{\text{in}} = 20.0\ \text{N}$

Unknown: AMA = ?
Original equation: AMA $= \dfrac{F_{\text{out}}}{F_{\text{in}}}$

Solve: AMA $= \dfrac{F_{\text{out}}}{F_{\text{in}}} = \dfrac{60.0\ \text{N}}{20.0\ \text{N}} = \mathbf{3.00}$

c. *Given:* AMA = 3.00
IMA = 4.00

Unknown: Eff = ?
Original equation: $\text{Eff} = \dfrac{\text{AMA}}{\text{IMA}}$

Solve: $\text{Eff} = \dfrac{\text{AMA}}{\text{IMA}} = \dfrac{3.00}{4.00} = 0.750 = \mathbf{75.0\%}$

Example 12: Clyde, a stubborn 3500-N mule, refuses to walk into the barn, so Farmer MacDonald must drag him up a 5.0-m ramp to his stall, which stands 0.50 m above ground level. a) What is the ideal mechanical advantage of the ramp? b) If Farmer MacDonald needs to exert a 450-N force on the mule to drag him up the ramp with a constant speed, what is the actual mechanical advantage of the ramp? c) What is the efficiency of the ramp?

Solution: For a ramp, ramp length is Δd_{in} and ramp height is Δd_{out}.

a. *Given:* $\Delta d_{\text{in}} = 5.0 \text{ m}$
$\Delta d_{\text{out}} = 0.50 \text{ m}$

Unknown: IMA = ?
Original equation: $\text{IMA} = \dfrac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}}$

Solve: $\text{IMA} = \dfrac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \dfrac{5.0 \text{ m}}{0.50 \text{ m}} = \mathbf{10.}$

b. *Given:* $F_{\text{out}} = 3500 \text{ N}$
$F_{\text{in}} = 450 \text{ N}$

Unknown: AMA = ?
Original equation: $\text{AMA} = \dfrac{F_{\text{out}}}{F_{\text{in}}}$

Solve: $\text{AMA} = \dfrac{F_{\text{out}}}{F_{\text{in}}} = \dfrac{3500 \text{ N}}{450 \text{ N}} = \mathbf{7.8}$

c. *Given:* IMA = 10.
AMA = 7.8

Unknown: Eff = ?
Original equation: $\text{Eff} = \dfrac{\text{AMA}}{\text{IMA}}$

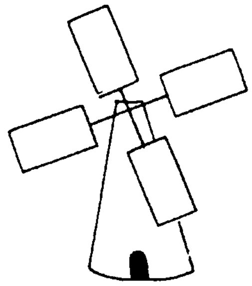
Solve: $\text{Eff} = \dfrac{\text{AMA}}{\text{IMA}} = \dfrac{7.8}{10.} = 0.78 = \mathbf{78\%}$

Practice Exercises

Exercise 11: Cathy, a 460-N actress playing Peter Pan, is hoisted above the stage in order to "fly" by a stagehand pulling with a force of 60. N on a rope wrapped around a pulley system. What is the actual mechanical advantage of the pulley system?

Answer: ______________________

Exercise 12: A windmill uses sails blown by the wind to turn an axle that allows a grindstone to grind corn into meal with a force of 90. N. The windmill has sails of radius 6.0 m blown by a wind that exerts a force of 15 N on the sails, and the axle of the grindstone has a radius of 0.50 m. a) What is the ideal mechanical advantage of the wheel? b) What is the actual mechanical advantage of the wheel? c) What is the efficiency of the wheel?

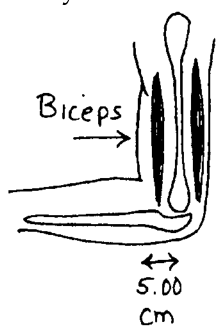


Answer: **a.** ______________________

Answer: **b.** ______________________

Answer: **c.** ______________________

Exercise 13: Winnie, a waitress, holds in one hand a 5.0-N tray stacked with twelve 3.5-N dishes. The length of her arm from her hand to her elbow is 30.0 cm and her biceps muscle exerts a force 5.0 cm from her elbow, which acts as a fulcrum. How much force must her biceps exert to allow her to hold the tray?



Answer: ______________________

Exercise 14: When building the pyramids, the ancient Egyptians were able to raise large stones to very great heights by using inclines. If an incline has an ideal mechanical advantage of 4.00 and the pyramid is 15.0 m tall, how much of an angle would the incline need in order for the Egyptian builder to reach the top?

Answer: ______________________

Exercise 15: The Ramseys are moving to a new town, so they have called in the ACME moving company to take care of their furniture. Debbie, one of the movers, slides the Ramseys' 2200-N china cabinet up a 6.0-m-long ramp to the moving van, which stands 1.0 m off the ground. a) What is the ideal mechanical advantage of the incline? b) If Debbie must exert a 500.-N force to move the china cabinet up the ramp with a constant speed, what is the actual mechanical advantage of the ramp? c) What is the efficiency of the ramp?

Answer: **a.** ______________________

Answer: **b.** ______________________

Answer: **c.** ______________________

11. 7.7
13. 280 N
15. a) 6.0
    b) 4.4
    c) 73%