Gravitational Force as a Centripetal Force

Newton's first law says that any body (planet, star, meteor etc.) will remain in motion unless acted upon by another unbalanced force. All planets and other celestial bodies exert gravitational forces on each other (this is how many of the planets were discovered). Because the sun is by far the most massive body in the solar system it exerts the most force and keeps the planets in their orbits. Here's how:



If there were no gravity an object would travel past the sun without changing its course as in A. Consider however if the object encountered the gravitational force, as in B. It would travel unaltered to point I, there the gravitational force exerts a force towards the center of the sun the object then "falls" toward the sun along the line from I to II. At point II the velocity of the object reenters the picture and the object moves perpendicular to the sun along the line labeled "no gravity". B shows a complete orbit executed in this manner. It is very choppy. In actuality both motions occur at the same time creating the circular (or elliptical) orbits we are familiar with.

Circular motion under gravitational force consititutes the basis for all satellite motion. Combining the equations for centripetal force and gravitational force we can calculate the velocities needed to keep satellites (in this case the moon) in orbit. Here's how:

$$F_{g} = F_{c}$$

$$Gm_{Earth}m_{moon} / r^{2} = m_{moon} v^{2}_{moon} / r$$

$$\begin{array}{l} Gm_{Earth} \ / \ r = v^2 \\ v = \sqrt{(Gm_{earth} \ / r)} \end{array}$$

Example:

If the moon is 3.84×10^5 km from the Earth, what must its speed be?

Reasoning: Convert r to meters 3.84×10^8 m

Solution:

$$v = \sqrt{(Gm_{earth}/r)}$$

v = $\sqrt{[(6.67 \text{ x}10^{-11} \text{Nm}^2/\text{kg}^2) \text{ x} (5.98 \text{ x} 10^{24} \text{kg}) / 3.84 \text{ x} 10^8 \text{m}]}$
v = 1.02 x 10³ m/s

Problems:

- 1. If you weigh 490N on earth how much do you weigh on the moon?
- 2. What is the gravitational force between two masses of 15kg each, when their centers are 0.25m? Could you detect this force with even sensitive equipment?
- 3. What would be the weight of the moon if it were resting on the surface of the earth? Remember that r is the distance between the centers of the objects. $r_{earth} = 6.37 \times 10^{6} \text{m} \text{ m}_{earth} = 5.98 \times 10^{24} \text{kg}.$
- 4. What is the mass of an object that weighs 55N on earth?
- 5. If a satellite were designed to orbit at 50.0km from the surface of the earth, what would be its velocity? How long would it take to complete 1 orbit?
- 6. What must the orbital speed of a satellite in an orbit that is 300km above the surface of the earth? How long would it take to complete 1 orbit?
- 7. The planet Neptune is 4.50×10^{12} km from the sun. The mass of the sun is 1.99×10^{30} kg. What is Neptune's orbital speed? How long does it take for Neptune to complete 1 orbit?