Physics Force Handout

Force	Any push or pull
Newton's 1st Law	Law of inertia (Restatement of Galileo's principle of **inertia**)
Newton's 2nd Law	$\sum F = ma$ (Note that this is the NET (total) force)
Newton's 3rd Law	Equal and opposite forces. For every action force there is an equal & opposite reaction force. Forces come in action - reaction pairs. When I kick a soccer ball the force on my foot is exactly equal and opposite as the force from my foot onto the ball regardless of the ball's mass.)

ΣF Key to all problems. ΣF_x ΣF in x direction on traditional coordinate axis. ΣF_y ΣF in y direction.

$\Sigma F_{//}$ ΣF parallel to a slope (direction of motion). $\Sigma F_{\perp}$ ΣF perpendicular to slope.

$\boxed{\sum F = ma}$ Sum of force is ***Net Force***. You may need to solve for ***a*** using the **kinematic equations**, then solve for force, or given force you solve for ***a*** and then use it in the **kinematic equations** to find ***v***, ***x***, or ***t***.

Strategy on force Problems:

1. Draw FBD.
2. Set direction of motion. ***What would the object do if it could?*** Considered this the positive direction.
3. Using the forces listed below write the ΣF equations relevant to the problem. ***In what direction is the problem moving?*** What matters, the ***x*** or the ***y*** direction? The ***parallel*** or the ***perpendicular*** direction? Any force vectors in the FBD pointing in the direction of motion are positive while any vectors the other way are negative.
4. Substitute known equation, (forces like F_g become ***mg***).
5. Substitute for ΣF. Ask yourself what the sum of force should be based on the chart below. Is the object standing still, moving at constant velocity, or accelerating. Substitute zero or ***ma*** for ΣF.

1	$v = 0$	$\Delta v = 0$	$a = 0$	$\Sigma F = 0$
2	v = +/- a constant value	$\Delta v = 0$	$a = 0$	$\Sigma F = 0$
3	v increasing or decreasing	Δv = +/- a constant value	a = +/- a constant value	$\Sigma F = ma$

6. Plug in and solve. (All values including 9.8 are entered as positives. The negative signs were decided when setting up the sum of force equation. Plugging in – 9.8 will just turn a vector assigned as $-F_g$ into a positive. You decided its sign based on the way it was pointing relative to the problems direction of motion. Don't reverse it now!)

F_P Push or Pull.

F_g Force of gravity. $\boxed{F_g = mg}$

F_T Tension is a rope, string, etc. This force has no equation. You either solve for it, or it cancels, or it's given.

F_N Force Normal. A contact force, always perpendicular to the surface. (On a tilted surface use $\Sigma F_{//}$ & $\Sigma F_{\perp}$)

F_{fr} Friction force. $F_{fr} = \mu F_N$ Always opposes motion. Static friction: not moving. Kinetic friction: object moving.

F_{ar} or F_D Force of air resistance. This force has no equation. You either solve for it, or it cancels, or it's given.

F_c Force Centripetal. It is the ΣF in circular motion problems. So F_c can be any force that keeps an object in circular motion. $F_c = F_g$ $F_c = F_N$ $F_c = F_T$ $F_c = F_{fr}$ $F_c = F_B$ *etc.*

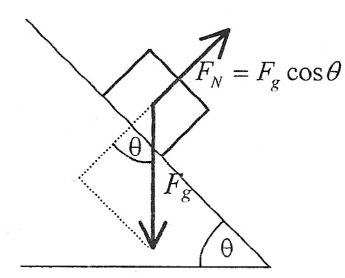
It can also be two or more of these added together. The direction of motion is toward the center. So any force directed toward the center is positive and any force directed outward is negative.

The key in using any of these equations is to ask yourself: **1.** What is causing the circular motion? **2.** Then set up the equality. **3.** Substitute known equations. **4.** Solve.

F_B Force due to a magnetic field. This force is perpendicular to the field and perpendicular to the velocity of the particle. So any charged particle will move in a circle. Use the right hand rule for positive charges or positive current, and use the left hand for negative charges or electron current.

$F_{\text{any subscript that make ssense to solve the problem}}$

Normal force: Gravity pulls the object **down** the slope **and into the slope**. If we consider only the motion into the slope (perpendicular), the object has no perpendicular velocity. So the $\Sigma F_{\perp} = 0$. Then the surface must push upward, equal and opposite to the perpendicular gravity component. Named the normal force, it is a contact force and operates perpendicular to any surface. It must counter only the component of gravity perpendicular to the surface.



$$F_N = F_g\cos\theta$$ Where θ is the angle between F_g and $F_{g\perp}$. It is also the tilt angle of the surface measured from the ground. Substituting ***mg*** for F_g.

$$F_N = mg\cos\theta$$ Flat surfaces $\theta = 0°$, $F_N = F_g$ *or* $F_N = mg$

Friction: opposes motion. Motion is always parallel to a surface, so friction always acts parallel.

Static Friction: Friction that will prevent an object from moving. As long as the object is standing still the force of friction must be equal to the push, pull, component of gravity or other force that attempts to move the object. (If there is no force attempting to cause motion, then there can be no friction).

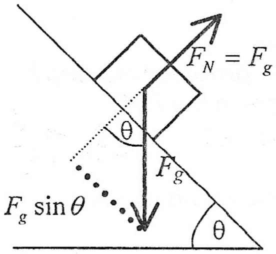
Static friction is the strongest type of friction since the surfaces have a stronger adherence when stationary.

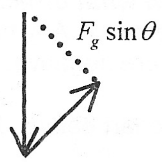
Kinetic Friction: Friction for moving objects. Once an object begins to move breaking static frictions hold, then the friction is termed kinetic. Kinetic friction is not as strong as static friction, but it still opposes motion.

Coefficient of friction: μ a value of the adherence or strength of friction. μ_k for kinetic friction and μ_s for static friction.

$$\boxed{F_{fr} = \mu F_N} \quad \text{so} \quad F_{fr} = \mu mg\cos\theta$$

Force Parallel (down the slope): Motion on a slope is parallel to the slope. F_g and F_N are at an angle to each other leaving a gap of magnitude $F_g \sin\theta$ when these two vectors are added tip to tail. $F_g \sin\theta$ is not a force by itself, it is the sum of force when F_g and F_N are added together. It is not part of the FBD. It describes the motion of the object parallel to the slope, if no other forces are acting on it. What if we sum the forces in the direction of motion (which is parallel to the slope)? $F_g \sin\theta$ is down the slope and positive, since objects generally want to go down hill (direction of natural motion is positive). Any force opposing the natural downward motion is a retarding force and is negative. So uphill is negative.



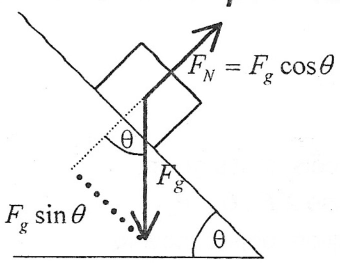


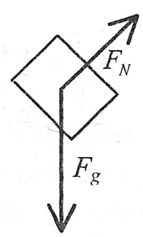
We need an overall sum of force in the **F||** direction.

$$\boxed{\Sigma F_{||} = F_g \sin\theta - F_{retarding}}$$

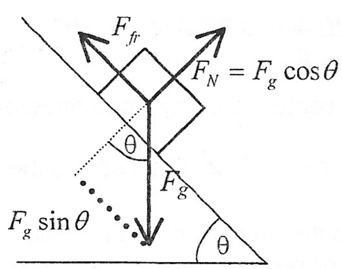
What do you use for force retarding? It could be friction F_{fr}, air resistance F_{ar}, a rope holding up the slope F_T, someone pushing up the slope F_P, or a combination of forces. Substitute the appropriate ***F*** and solve.

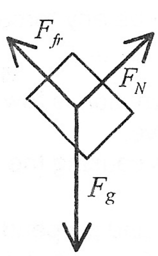
Friction on the slope: Friction is the retarding force in the scenarios discussed above.



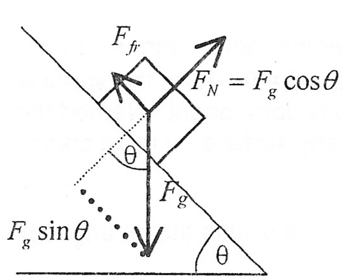


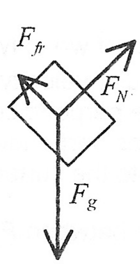
1. No friction.
   (What will the object do? Accelerate $\Sigma F = ma$)
   $$\Sigma F_{||} = F_g \sin\theta - F_{retarding}$$
   $$\Sigma F_{||} = F_g \sin\theta - 0$$
   $$ma = mg\sin\theta$$
   $$a = g\sin\theta$$





2. ***v = 0*** or ***v*** is **constant**.
   (No acceleration $\Sigma F = 0$)
   $$\Sigma F_{||} = F_g \sin\theta - F_{retarding}$$
   $$0 = F_g \sin\theta - F_{fr}$$
   $$\mu mg\cos\theta = mg\sin\theta$$
   $$\mu\cos\theta = \sin\theta$$
   $$\mu = \tan\theta$$



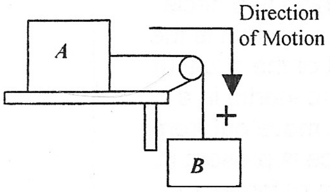


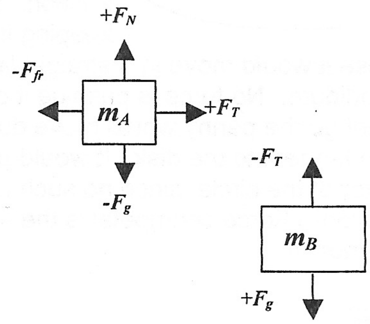
3. Accelerating with friction present.
   (Accelerates so $\Sigma F = ma$)
   $$\Sigma F_{||} = F_g \sin\theta - F_{retarding}$$
   $$\Sigma F_{||} = F_g \sin\theta - F_{fr}$$
   $$ma = mg\sin\theta - \mu mg\cos\theta$$
   $$a = g\sin\theta - \mu g\cos\theta$$
   $$a = g(\sin\theta - \mu\cos\theta)$$

Complex Force Problems

Set direction of motion as positive. If you are not sure what the direction of motion will be take a guess. If the problem returns negative values for the final result, you were wrong, the problem went the opposite of your prediction.

Vertical & Horizontal





Tension is the same for both blocks. Rearrange to get equations in terms of tension, then set them equal so tension cancels. Then substitute and solve.

$$\sum F_A = F_T - F_{frA} \qquad \sum F_B = F_{gB} - F_T$$

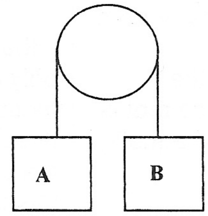
$$F_T = \sum F_A + F_{frA} \qquad F_T = F_{gB} - \sum F_B$$

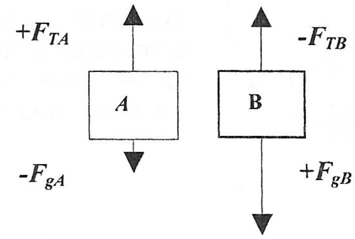
$$\sum F_A + F_{frA} = F_{gB} - \sum F_B$$

$$m_A a + \mu m_A g \cos\theta = m_B g - m_B a$$

$$a = \frac{m_B g - \mu m_A g \cos\theta}{m_A + m_B}$$

Pulley





If it doesn't say which is more massive, pick one. In this case I picked ***B*** as the heavier object and used this to set the direction of motion. Find what does not change, ***T***, and rearrange in terms of this. Set the equations as equal, substitute and solve.

$$\sum F_A = F_T - F_{gA} \qquad \sum F_B = F_{gB} - F_T$$

$$F_T = \sum F_A + F_{gA} \qquad F_T = F_{gB} - \sum F_B$$

$$\sum F_A + F_{gA} = F_{gB} - \sum F_B$$

$$m_A a + m_A g = m_B g - m_B a$$

$$a = \frac{m_B g - m_A g}{m_A + m_B}$$

Friction on horizontal surfaces

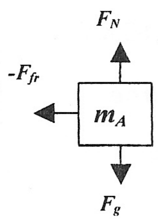
1. Friction is the only force in the horizontal direction.

$$\Sigma F = F_{fr}$$

$$\Sigma F = \mu F_g$$

$$ma = \mu mg$$

$$a = \mu g$$



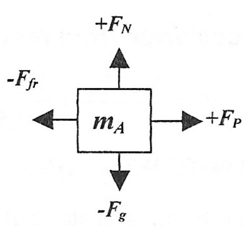
2. When friction and the forward force are equal. Object can be standing still or moving at constant velocity.

$$\Sigma F = F_P - F_{fr}$$

$$0 = F_P - F_{fr}$$

$$F_P = F_{fr}$$

$$F_P = \mu mg$$

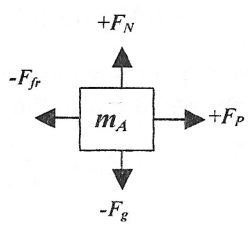


3. When friction is not strong enough to prevent the object from accelerating anyway.

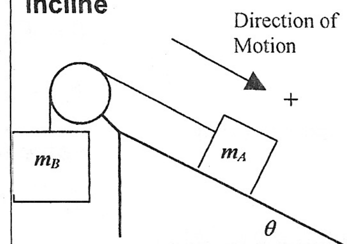
$$\Sigma F = F_P - F_{fr}$$

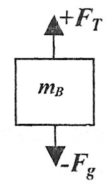
$$ma = F_P - \mu mg$$

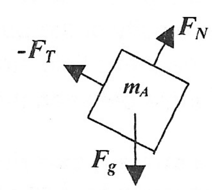
$$a = \frac{F_P - \mu mg}{m}$$



Incline







I picked m_A as moving down the slope, so m_B moves up. Tension prevents m_A from sliding down the slope and is therefore acting like friction. If there was friction it would be another arrow opposing motion down the slope. Just subtract it as well. F_g and F_N are at angles to each other leaving a vector gap of $F_g\sin\theta$ (see previous page)

$$\sum F_{\parallel} = F_g \sin\theta - F_{retarding}$$

$$\sum F_A = F_{gA}\sin\theta - F_T \qquad \sum F_B = F_T - F_{gB}$$

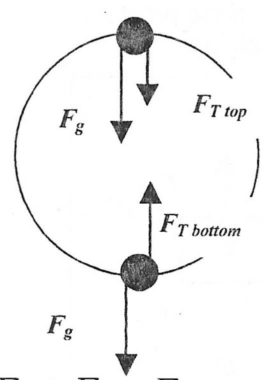
$$F_T = F_{gA}\sin\theta - \sum F_A \qquad F_T = \sum F_B + F_{gB}$$

$$F_{gA}\sin\theta - \sum F_A = \sum F_B + F_{gB}$$

$$m_A g \sin\theta - m_A a = m_B a + m_B g$$

$$a = \frac{g\left(m_A \sin\theta - m_B\right)}{\left(m_A + m_B\right)}$$

Vertical Circular Motion



A ball at the end of a string is swung in a vertical circle. Any force pointing to the center is positive centripetal force, while force vectors pointing away from the center are negative centripetal force. Sum the forces. Look for the force that is the same, and set up an equality.

$$F_c = F_g + F_{T\,top}$$

$$F_{T\,top} = F_c - F_g$$

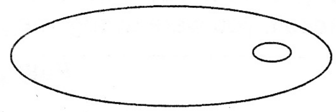
$$F_{T\,top} = m\frac{v^2}{r} - mg$$

$$F_c = -F_g + F_{T\,bottom}$$

$$F_{T\,bottom} = F_c + F_g$$

$$F_{T\,bottom} = m\frac{v^2}{r} + mg$$

Horizontal Circular Motion



A penny on a circular disk rotating horizontally. What keeps it from flying off? Friction. Something must be keeping it going in a circle. Otherwise it would move in a straight line. Friction is the only candidate. No force is pushing it out of the circle (If friction let go the penny would move due to inertia in a direction tangent to the disk. It would not move out from the center of the circle, since no such force is present in this problem.) Force centripetal is the sum of forces for circular motion.

$$F_c = F_{fr}$$

$$m\frac{v^2}{r} = \mu mg$$

$$v = \sqrt{\mu gr} \quad \text{or} \quad \mu = \frac{v^2}{rg}.$$

Magnetic Field

Force on a charged particle

A charged particle moving in a magnetic field will experience a force causing it to follow a curved path and be deflected from its original course. If the force is strong enough the particle can be made to follow a circular path.

$$F_B = qvB\sin\theta$$

q is the charge on the particle. See constants table.
v is the velocity of the particle.
B is the magnetic field strength.
θ is the angle between the velocity and magnetic field.

Force on a current carrying wire

The magnetic field can also move a current carrying wire. The wire can jump.

$$F_B = BI\ell\sin\theta$$

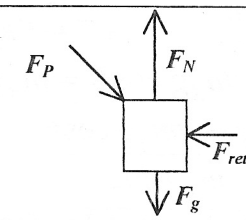
B is the magnetic field strength.
I is the current in the wire.
ℓ is the charge on the particle
θ is the angle between the velocity and magnetic field.

The **Right Hand Rule** is used to determine the direction of deflection of the charged particles in the top scenario and the direction of movement of the wire in the bottom scenario.

How do you choose the right equation?

q is for charged particles, and ℓ length of wire.

Lawn Mower



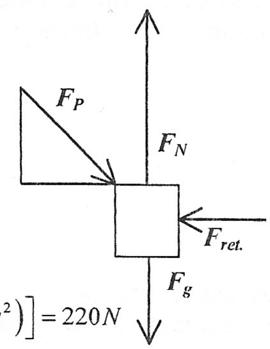
Pushing with 90 *N* at 45°
Constant speed. $\sum F = 0$

Solve for the Retarding Force

$$\sum F_x = F_{Px} - F_{ret.}$$

$$0 = F_{Px} - F_{ret.}$$

$$F_{ret.} = F_{Px} = 90N\cos 45^o = 63.6N$$



Solve for the Normal Force

$$\sum F_y = -F_{Py} + F_N - F_g$$

$$F_N = \sum F_y + F_{Py} + F_g$$

$$F_N = 0 + (90N\sin 45^o) + [(16kg)(9.8\,m/s^2)] = 220N$$

Solve for F_p to accelerate from rest to 1.5 *m/s* in 2.5 *s*

$$v_x = v_{x_o} + a_x t \qquad a_x = \frac{v_x - v_{x_o}}{t} = \frac{1.5-0}{2.5} = 0.6\,m/s^2$$

$$\sum F_x = ma = (16kg)(0.6\,m/s^2) = 9.6N$$

You need this force to accelerate, but you still need to overcome the retarding force.

$$\sum F_x = F_{Px} - F_{ret.}$$

$$F_{Px} = \sum F_x + F_{ret.} = 9.6N + 63.6N = 73.2N$$

But you aren't pushing in the *x* direction. You need the push at 45° to generate 73.2 *N* in the *x* direction.

$$F_{Px} = F_{push}\cos 45^o \qquad F_{push} = \frac{F_{Px}}{\cos 45^o} = \frac{73.2N}{0.707} = 104N$$