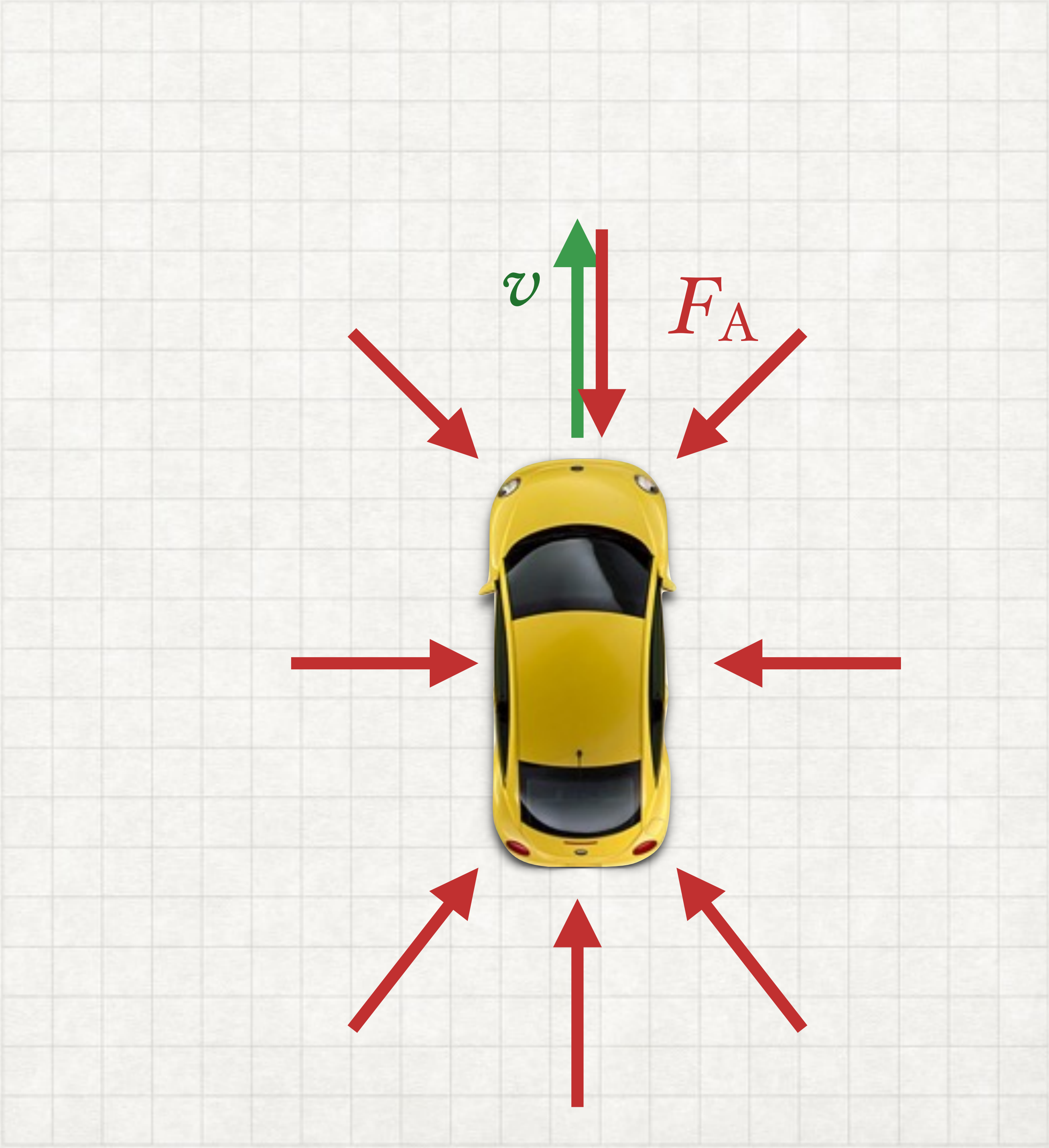
"WE COME SPINNING OUT OF NOTHINGNESS, SCATTERING STARS LIKE DUST" – JALAL AD-DIN RUMI

CIRCULAR MOTION

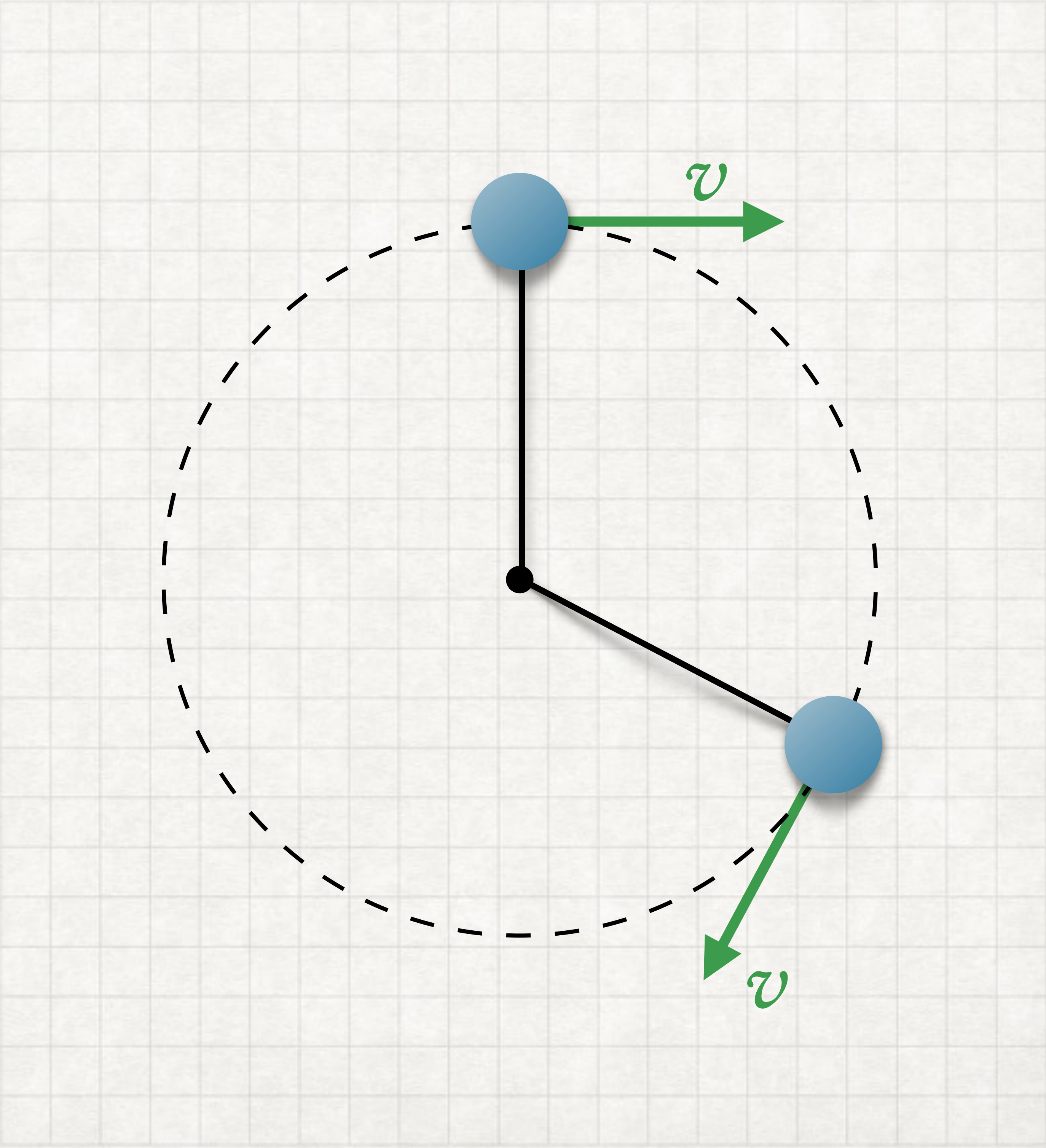
HOW DO YOU MAKE AN OBJECT TURN?

- 1st Law: An object in motion will tend to stay in motion at a *constant speed* in a *straight line* unless acted on by an external, unbalanced force.
- Ok, so we need an external, unbalanced force
  - But where should we apply it?

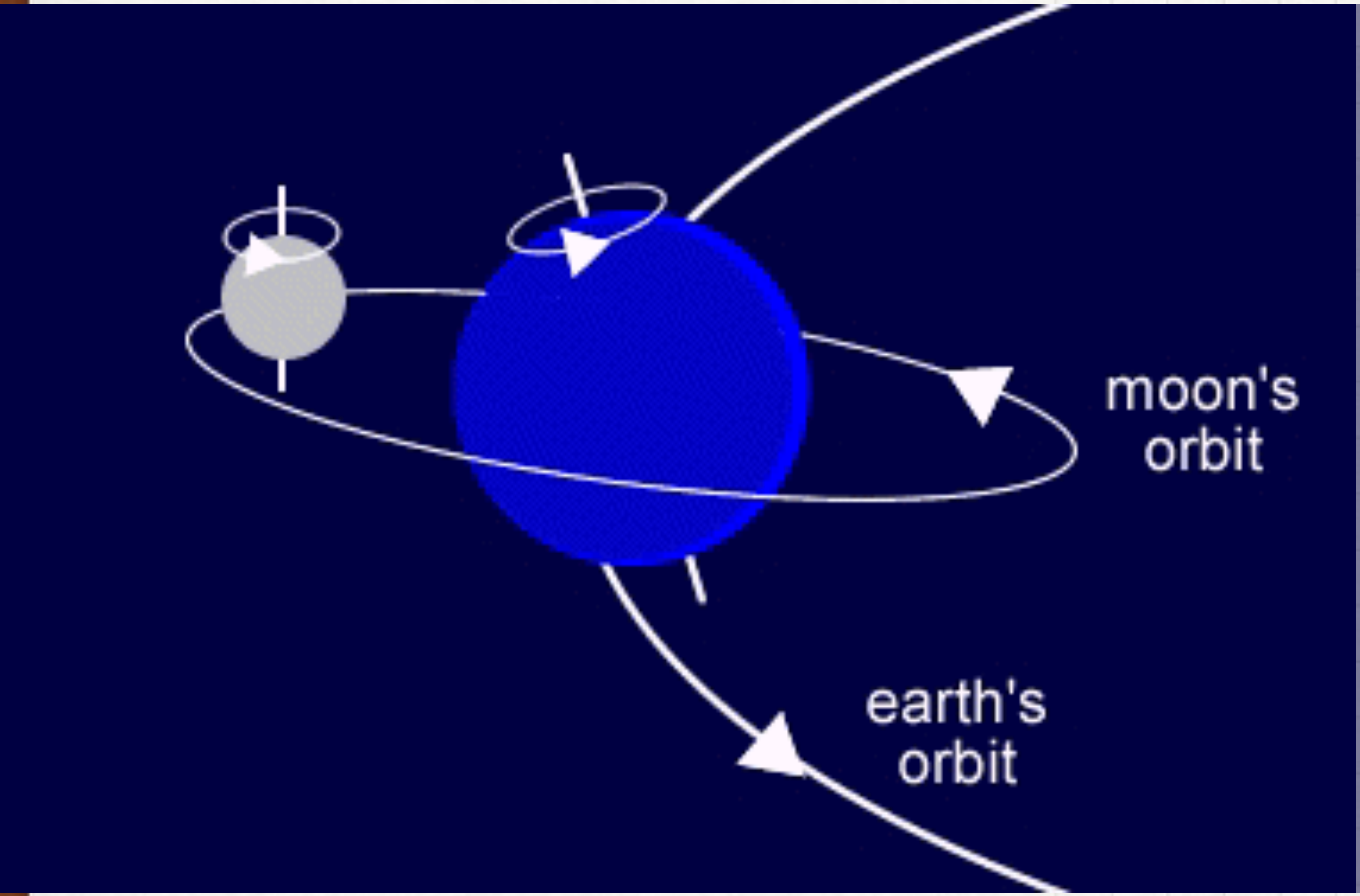


UNIFORM CIRCULAR MOTION

- An object that moves in a circle at a constant speed v is said to experience uniform circular motion
- The *magnitude* of the velocity remains constant, but the *direction* is continuously changing



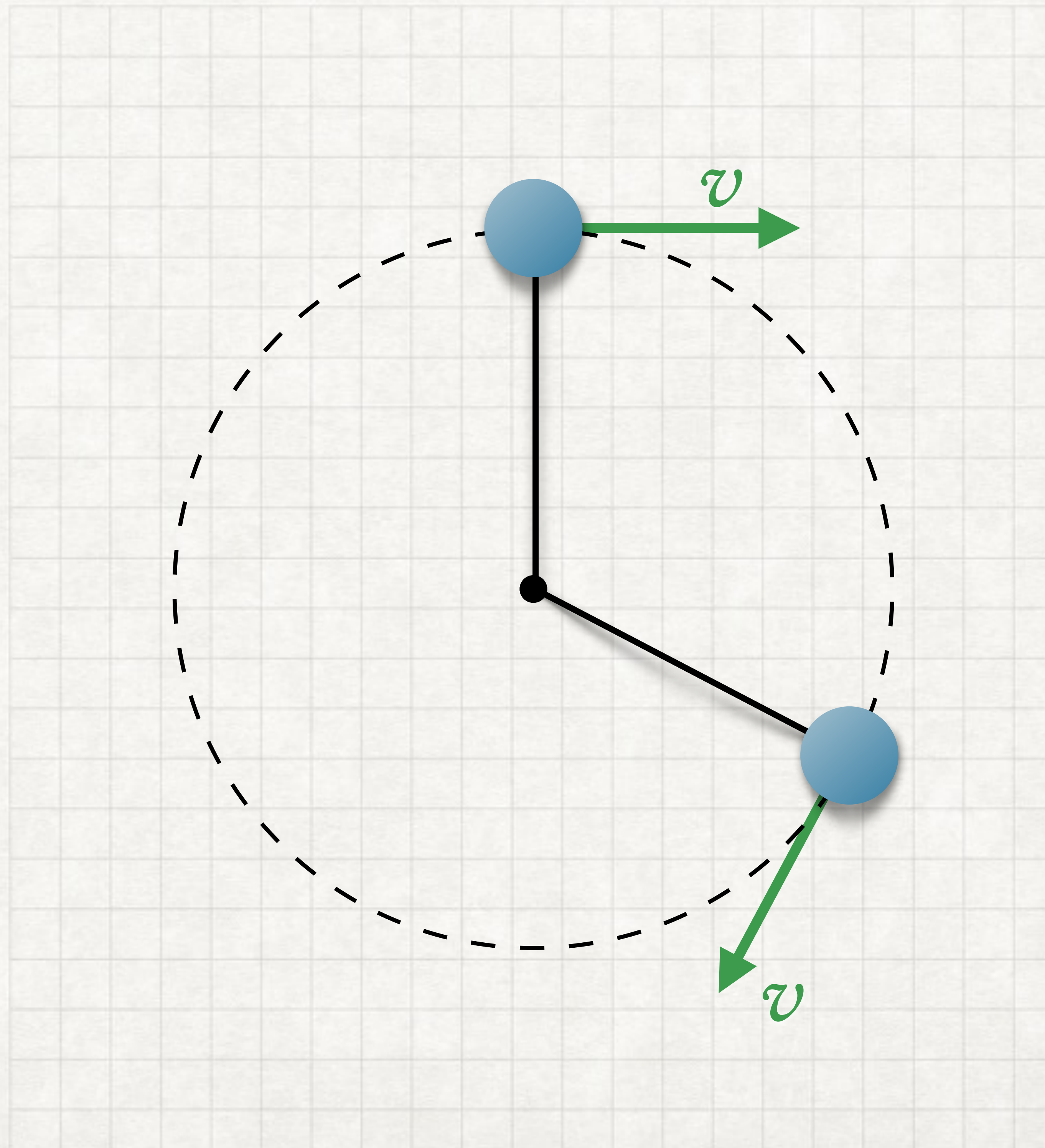
UNIFORM CIRCULAR MOTION



- Rotation (or *spin*) — when a body turns about an *internal* axis
- Revolution — when a body turns about an *external* axis
  - The Earth *rotates* around an axis passing through its geographic poles once every 24 hours
  - The Earth *revolves* around the Sun once every 365.25 days

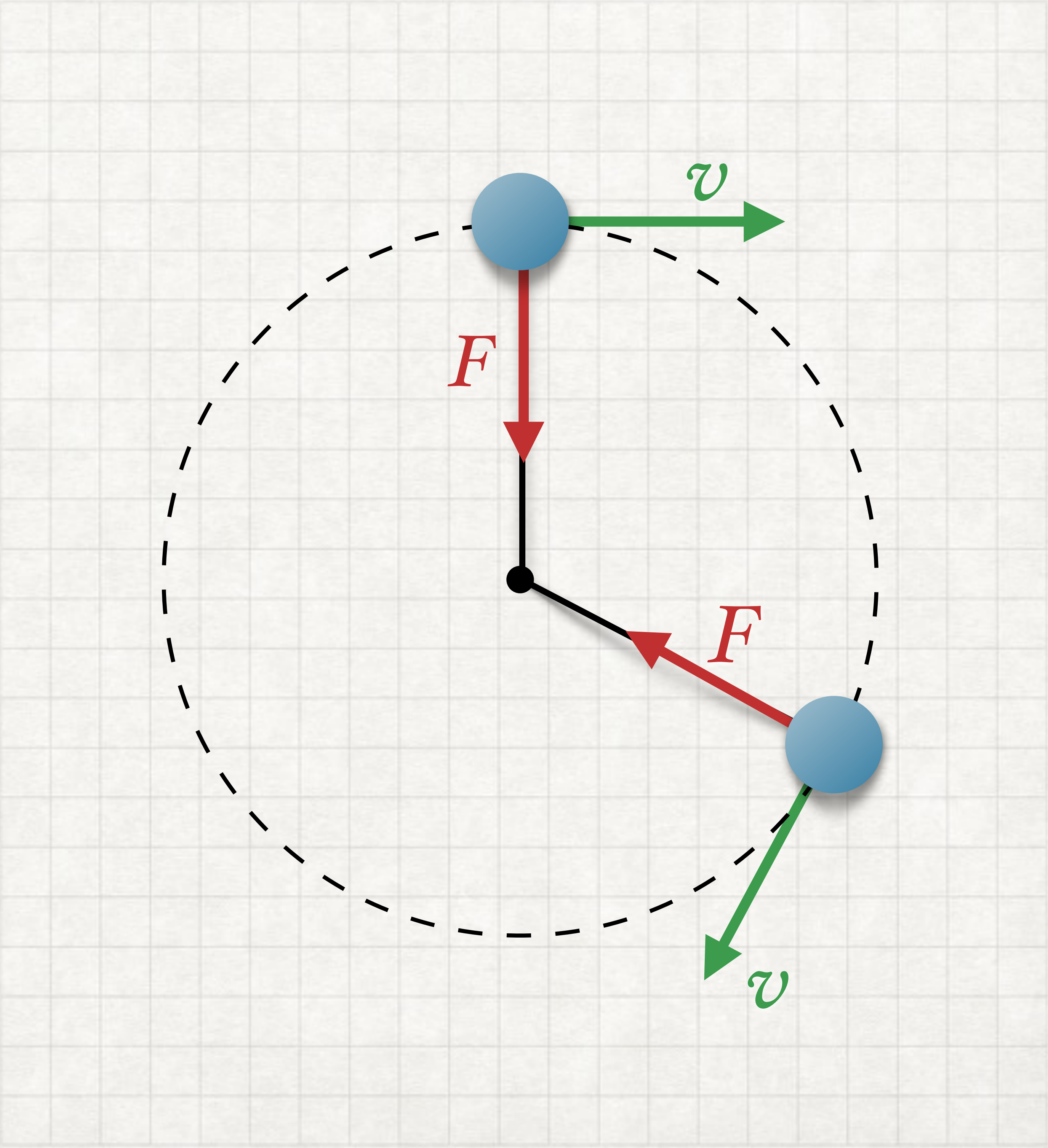
UNIFORM CIRCULAR MOTION

- radial - behavior toward and away from the center of the circle
- tangential - behavior along the edge of the circle
  - e.g. v is the tangential velocity (v_{t})



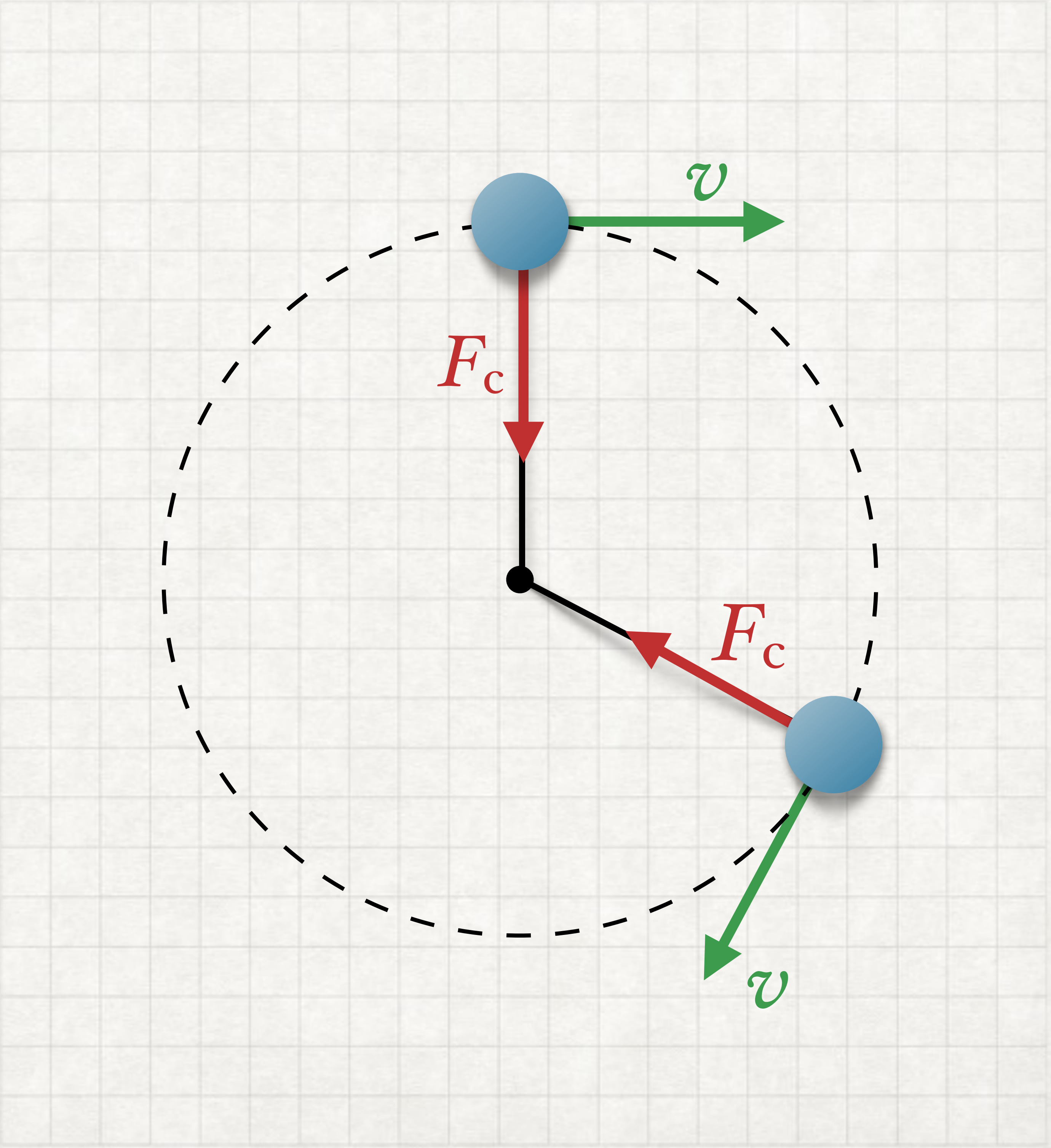
UNIFORM CIRCULAR MOTION

- Which way must the force be acting to make the object move in a circle?
- Forces which point toward the center of rotation are called centripetal forces (F_c), meaning "center seeking" forces



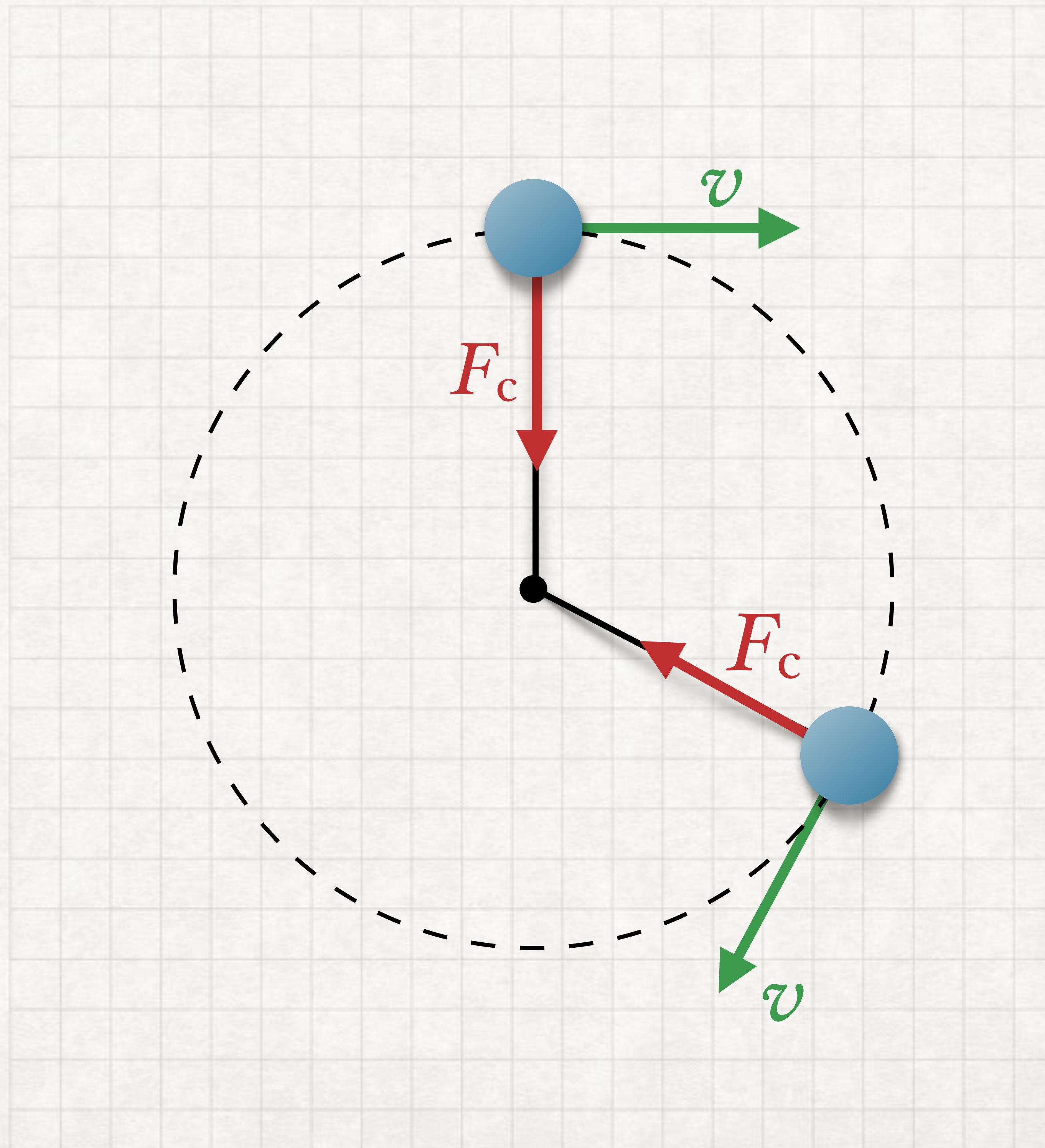
CENTRIPETAL FORCE

- What forces act as centripetal forces?
  - ball on a string swung in a circle: *tension*
  - car rounding a bend: *friction*
  - orbit of planets: *gravity*
  - rollercoaster car going around a loop: *normal force*



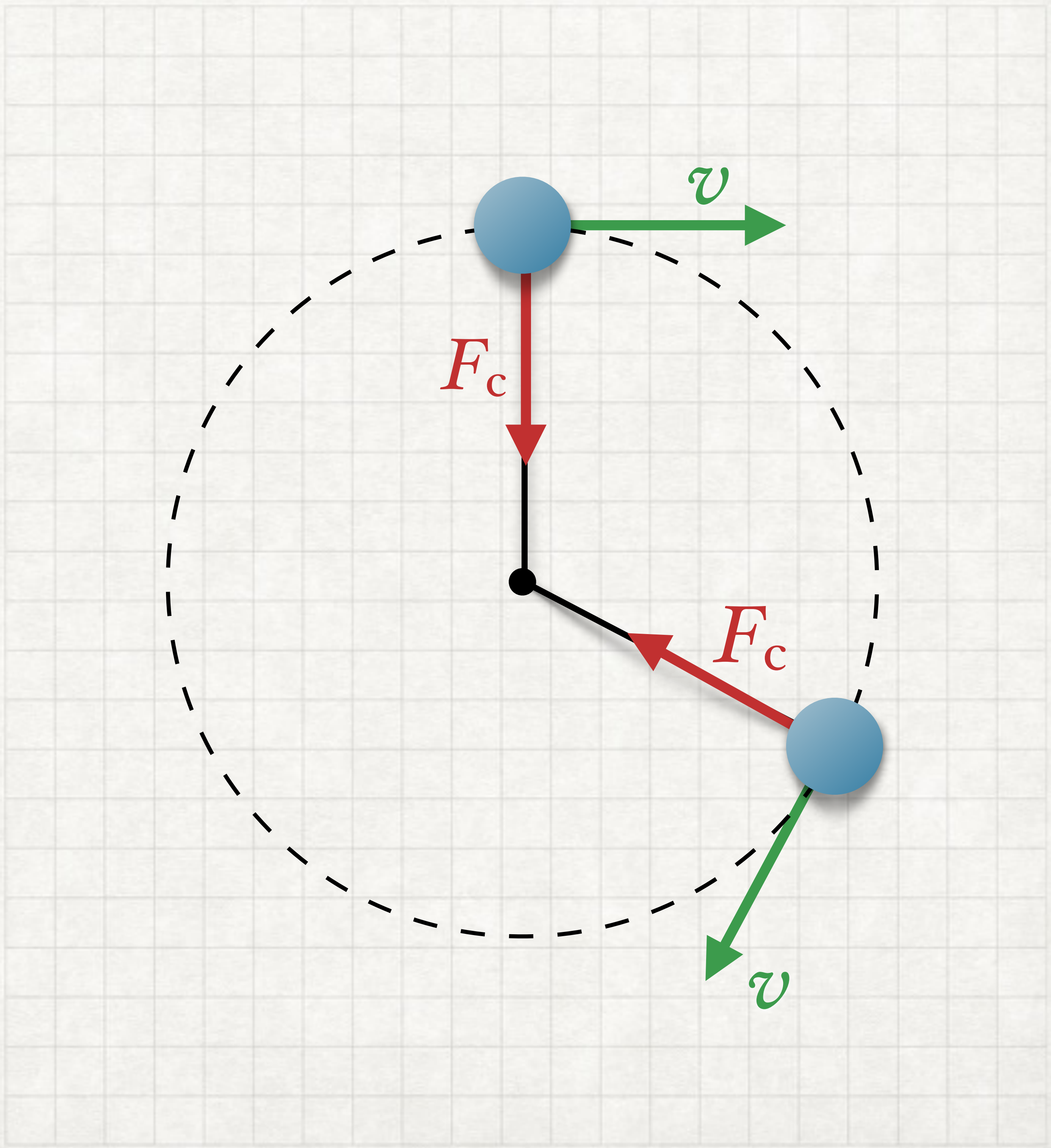
CENTRIPETAL FORCE

- As I swing a ball in a circle overhead, three quantities determine how big a force I'll need to keep it in a circle:
  - How big is the ball: m
  - How big is the circular path: r
  - How fast is the ball moving through the circle: v



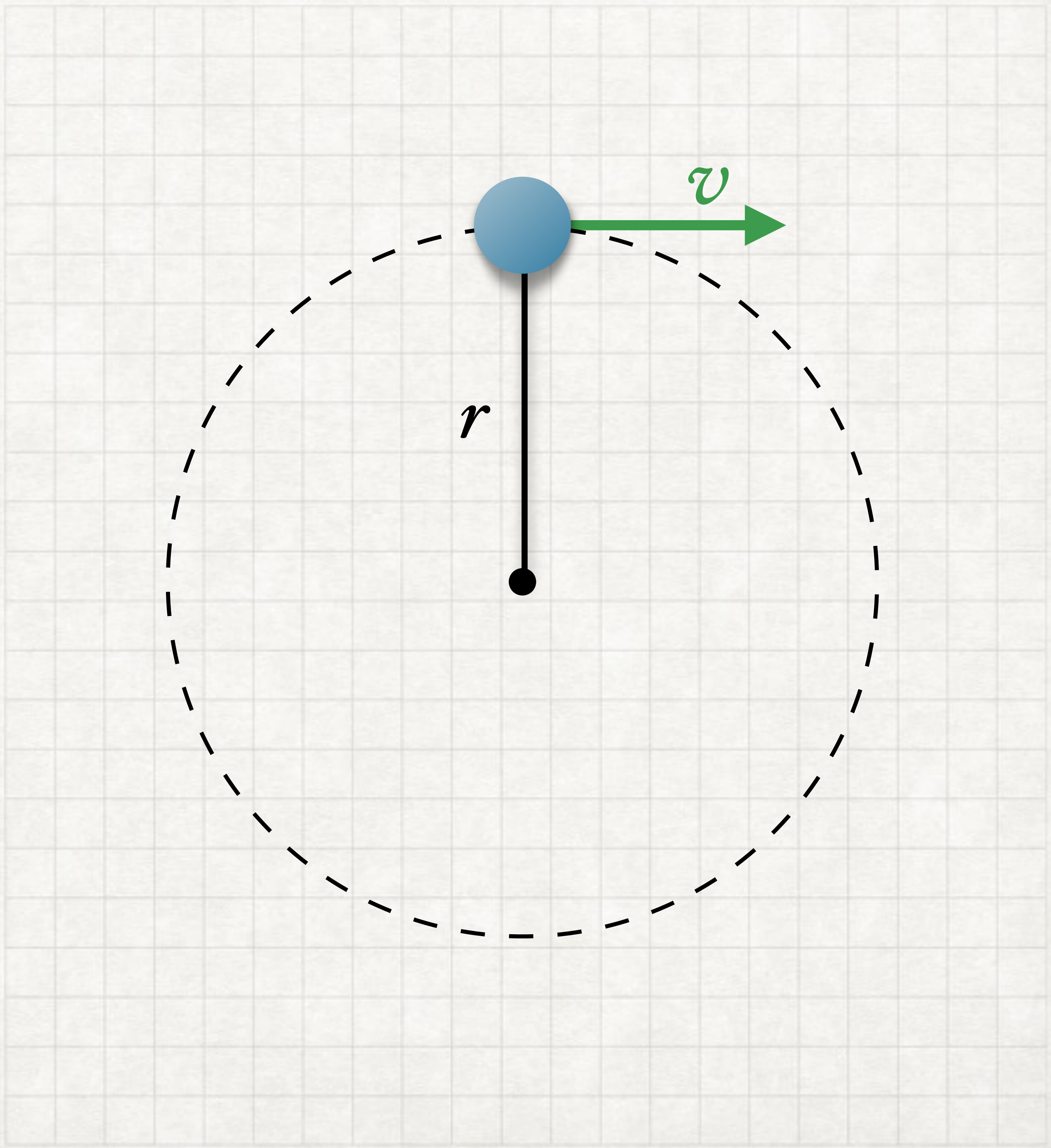
CENTRIPETAL FORCE

- $F_c = ma_c$
- $a_c = v^2/r$
  - (Proof: https://www.youtube.com/watch?v=TNX-Z6XR3gA)
- $F_c = mv^2/r$



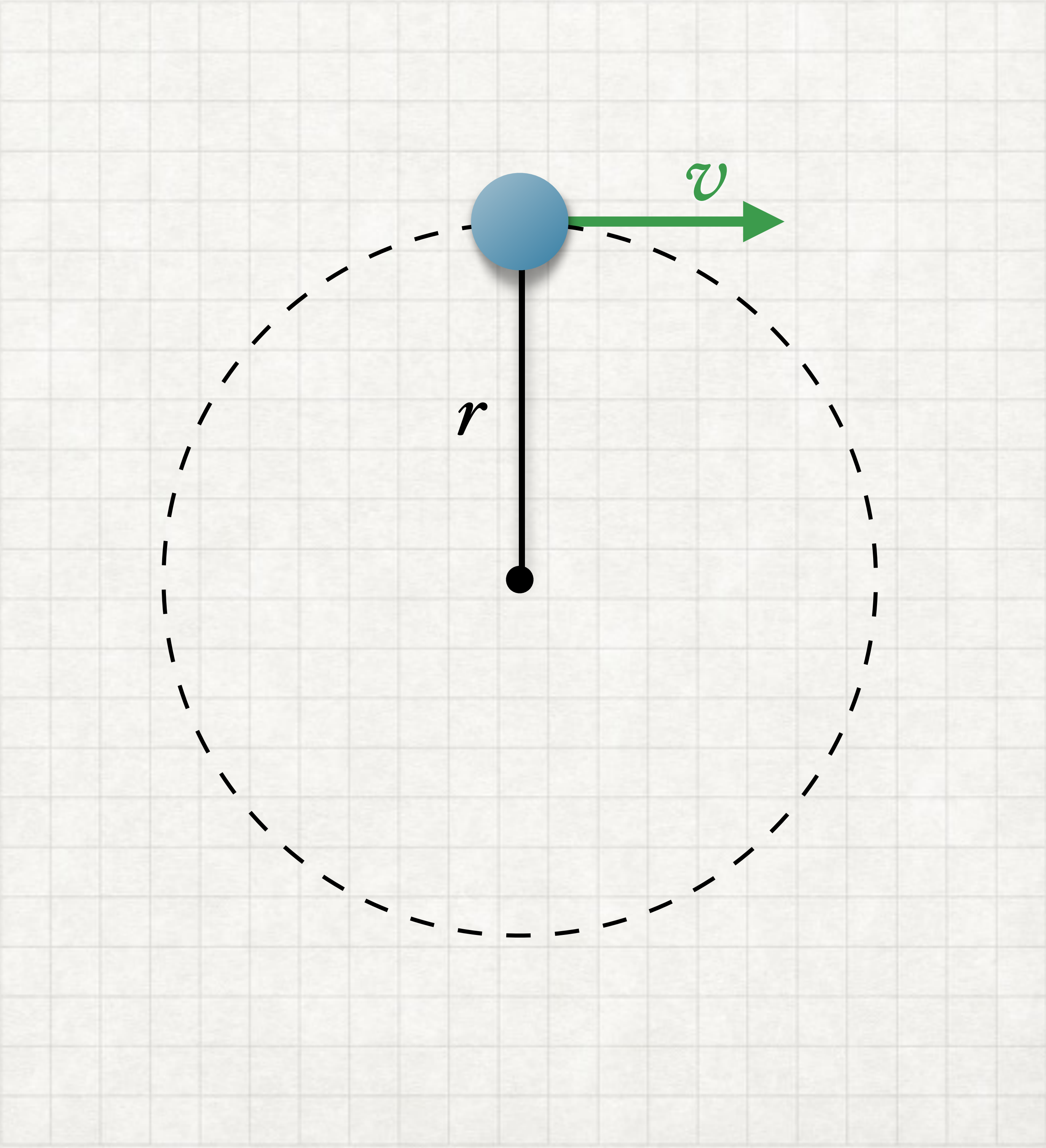
FREQUENCY & PERIOD

- Frequency (f) is the number of revolutions per second
  - measured in Hertz (Hz)
  - 1 Hz = 1 rev/s
- Period (T) is the time required to make one full revolution
  - measured in seconds
  - $T = 1/f$



FREQUENCY & PERIOD

- You swing a ball from a sting of length *r* and, using a stopwatch, measure a period of *T*
- How fast must you be swinging the ball?
  - $v = 2\pi r/T$



EXAMPLE 1

- You got the Ball and Chain! Set it to y or x and swing to do serious damage.
- You can swing the 18.0 kg ball from its 1.50-m-long chain through 2.00 revolutions per second. What is the force of tension in the chain?
  - Ans. F_T = 4,260 N

EXAMPLE 2

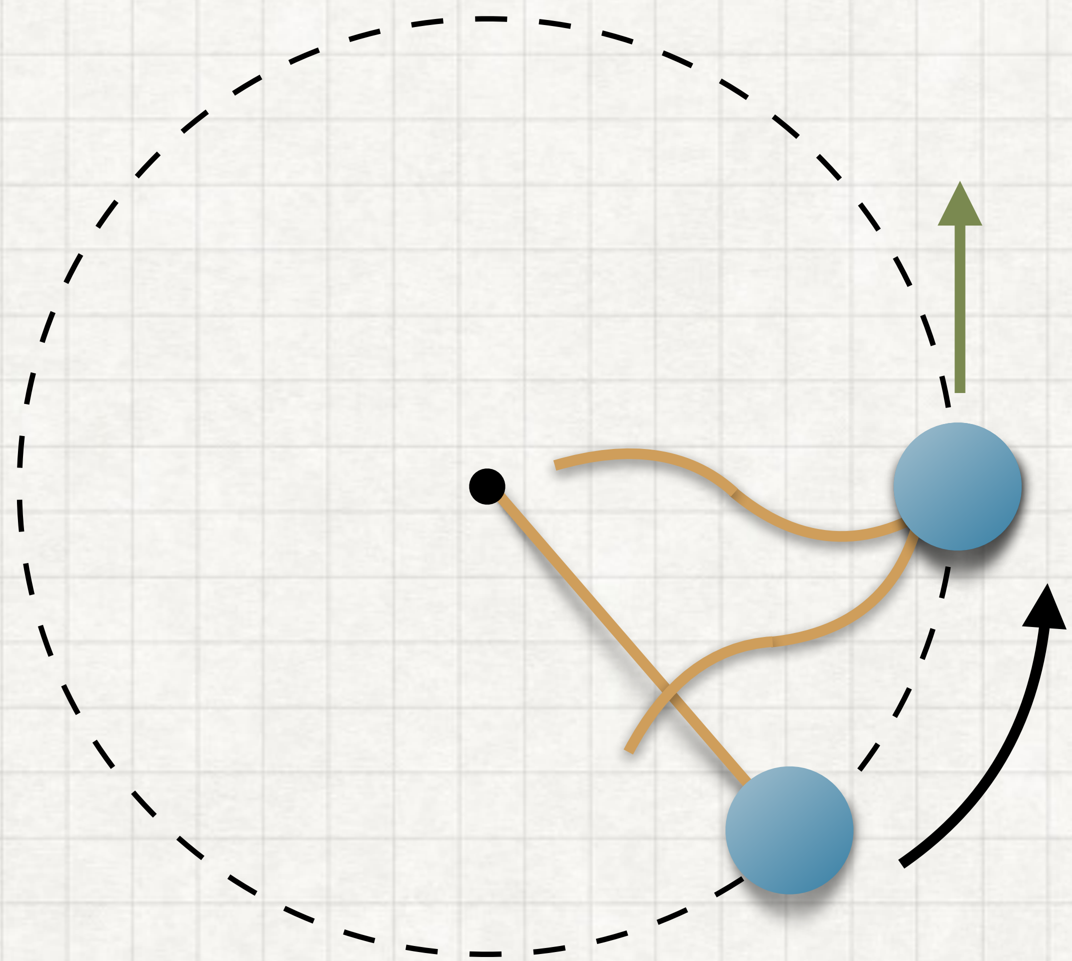
- The Moon's nearly circular orbit about the Earth has a radius of about 384,000 km and a period *T* of 27.3 days. Determine the acceleration of the Moon towards the Earth.
  - Ans. $a_c = 2.72\times10^{-3}$ m/s^2

SANITY CHECK

- You're driving along in your car when you make a hard and quick left
- In what direction is the (net) force acting on your body?
  - *To the left!*
- But your body *feels* like it's being pushed to the right because your inertia resists changes to its motion
  - The so-called centrifugal ("center fleeing") force is a fictitious force experienced because your reference frame (the car, in this case) is accelerating

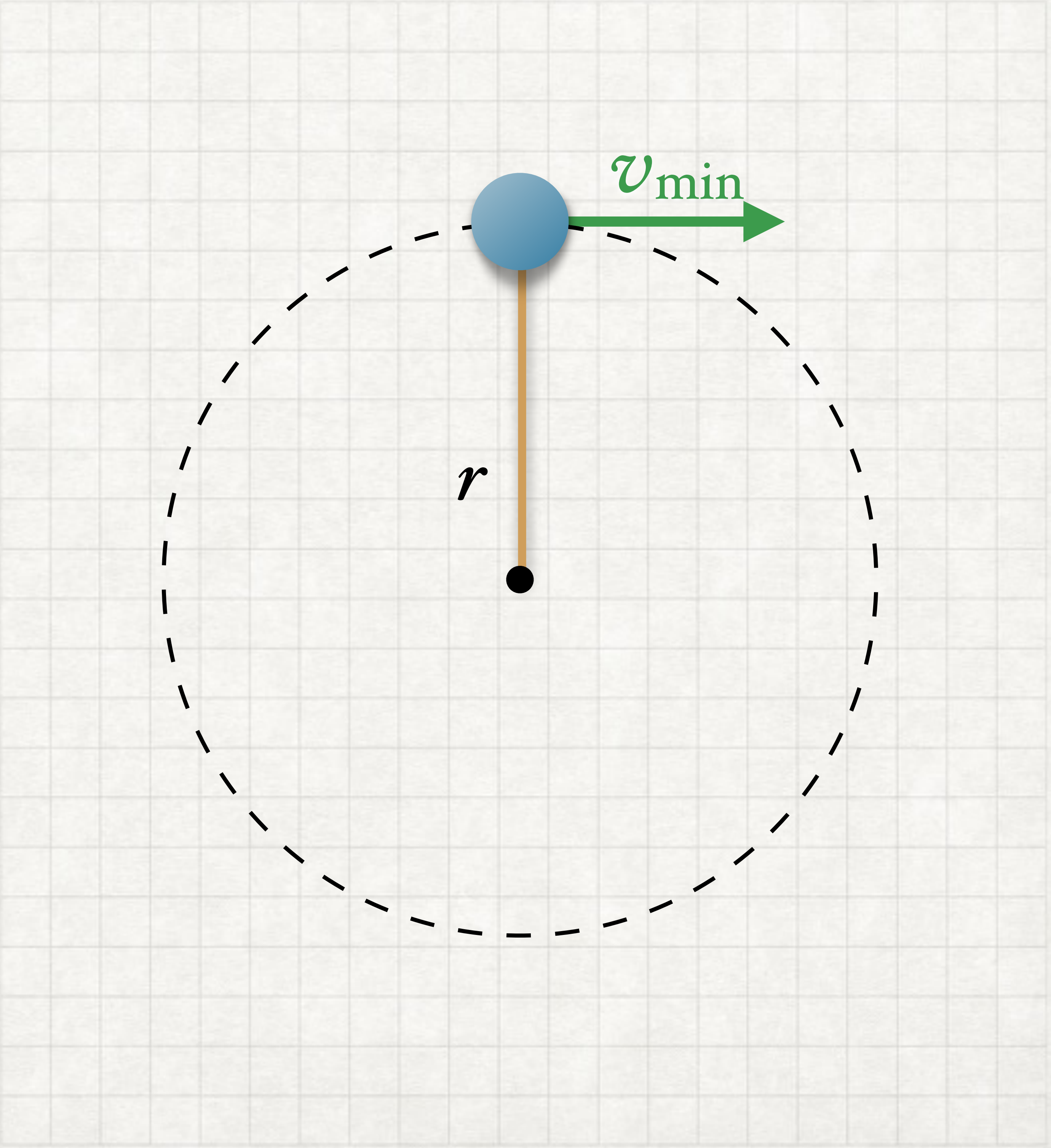
SANITY CHECK

- A ball is swung in a horizontal circle and released as shown to the right. Which way will the ball travel once it's released?
- *On a tangential path*



EXAMPLE 3

- A 0.150 kg ball on the end of a 1.10-m-long cord (negligible mass) is swung in a *vertical* circle. Determine the minimum speed the ball must have at the top of its arc so that it continues moving in a circle.
  - *Ans.* $v_{min} = 3.28$ *m/s*



SANITY CHECK

- A rider on a Ferris wheel moves in a vertical circle of radius *r* at constant speed *v*. Is the normal force the seat exerts on the rider at the top of the ride greater than, less than, or equal to the normal force the seat exerts at the bottom?
  - *Ans.* $F_{N,top} < F_{N,bottom}$

EXAMPLE 4

- A 1000-kg car rounds a curve on a flat road of radius 50 m at a speed of 50 kph. Will the car make the turn, or will it skid if:

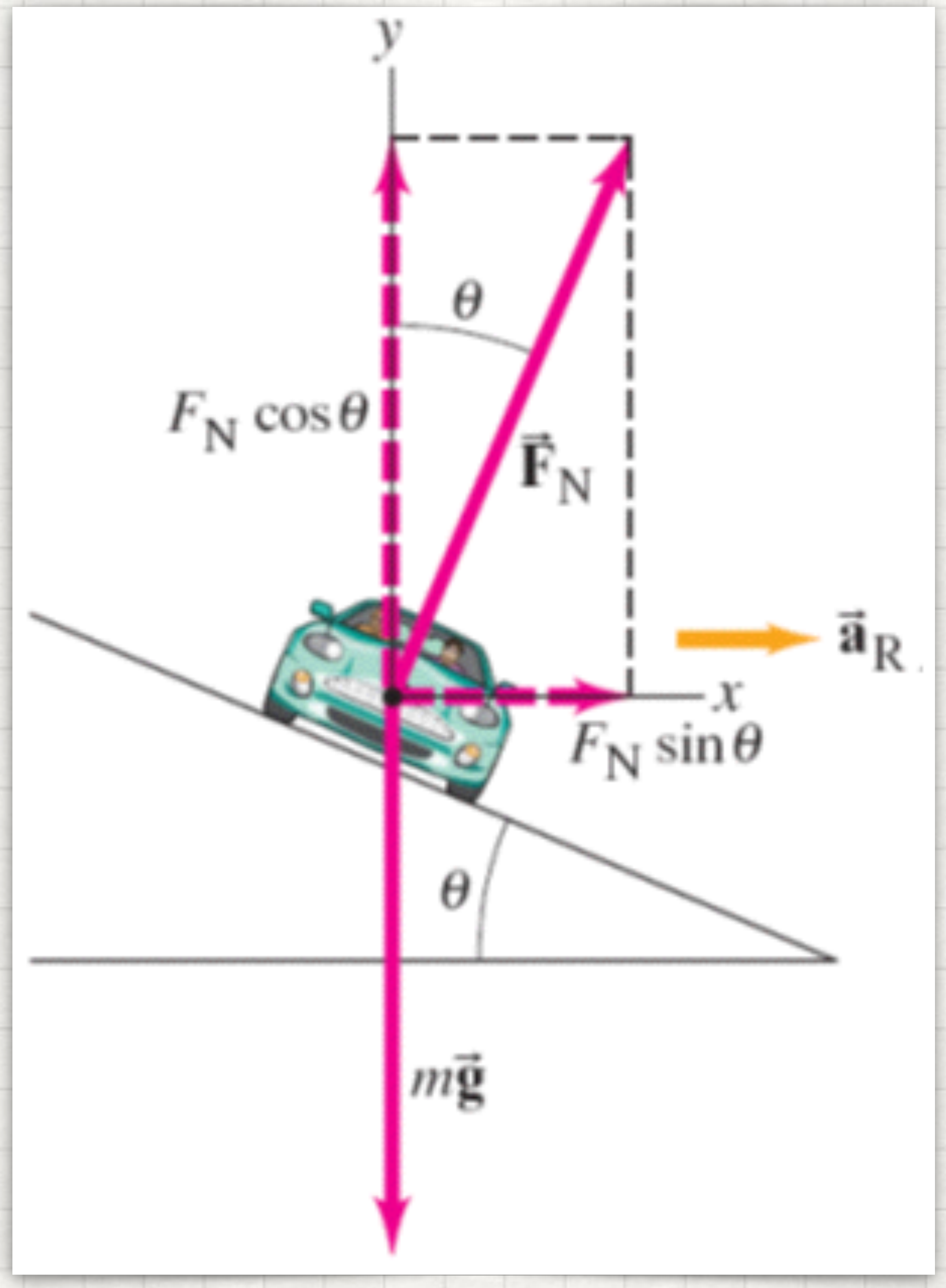
  a) the pavement is dry and $\mu_s = 0.60$?

  b) the pavement is icy and $\mu_s = 0.25$?

- *Ans.* $F_c = 3900\ N$
- *a)* $F_{fr,max} = 5900\ N$*; yes*
- *b)* $F_{fr,max} = 2500\ N$*; skid*

BANKED CURVES

- The banking of curves can reduce the chance of skidding because the normal force will have a component toward the center, thus reducing the reliance on friction
- For a given banking angle, **θ**, there exists one speed for which no friction at all is required
  - that speed is called "design speed"
- When a car travels at the road's design speed, $F_N \sin\theta = mv^2/r$



EXAMPLE 5

- At what angle should a civil engineer design the bank of an expressway off-ramp curve of radius 50. m for a design speed of 50. kph?
  - *Ans.* θ *= 22°*