BC Calculus –Review Sheet

When you see the words ….	This is what you think of doing…
1. Find the area of the unbounded region represented by the integral $\int_1^{\infty} f(x)dx$ (sometimes called a horizontal improper integral).	Set up $\lim_{b\to\infty}\int_1^b f(x)\,dx$ to see if the area diverges or converges.
2. Find the area of a different unbounded region under f(x) from (a,b], where $\lim_{x\to a^+} f(x)=\infty \text{ or } -\infty$, where the area is represented by $\int_a^b f(x)dx$,(sometimes called a vertical improper)	Set up $\lim_{x\to a^+}\int_a^b f(x)\,dx$ to see if the area diverges or converges.
3. Given a f(x), find arc length of the function on the interval (a, f(a)) and (b,f(b)).	Use the integral: $L=\int_a^b \sqrt{1+\left[f'(x)\right]^2}dx$.
4. Given a curve in parametric form where $x=f(t), y=g(t)$, find the arc length of the curve on the interval $[t_1,t_2]$.	Use the integral: $L=\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt$
5. Given $\frac{dy}{dx}=F(x,y)=xy$ and an initial point $(x_o,y_o)=(1,1)$, find an approximate value for $f(1.2)$ and $\Delta x=0.1$	Think about Euler's Method to draw tangent lines and approximate along the tangent lines. First calculate the slope at (1,1) and write an equation of a tangent line to f at (1,1). Use this line to approximate a new point at x=1.1 using $\Delta x=0.1$. This gives you a second point to repeat the procedure again. Write another tangent line with a new slope and approximate the value of f(1.2) by moving along this second tangent line to the point x = 1.2.
6. Given the differential equation of the form $\frac{dP}{dt}=kP\left(1-\frac{P}{L}\right)$ for P as a function of t, where k and L are constants.	Separate the differentials, use partial fractions, integrate, use an initial condition to solve for the constant and end up with an equation of the form: $P=\frac{L}{1+Ae^{-Mkt}}$

7. Given the differential equation $\frac{dP}{dt} = 12P - 4P^2$ where P is measuring the number an animal present on day 0. Find the value of P when the number of these animals is increasing the fastest.	7. First notice that $\frac{dP}{dt} = 12P - 4P^2$ is a parabola, so rewriting it in the form $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$ or $\frac{dP}{dt} = 12P\left(1 - \frac{P}{3}\right)$ tells us that the $\frac{dP}{dt} = 0$ v when P=0 or P=3. The number of animals is increasing fastest at the midpoint of 0 and 3 or 1.5.
8. Given the differential equation $\frac{dP}{dt} = 1200P - 400P^2$ where P is measuring the number an animal present on day 0. Determine the $\lim_{n \to \infty} P(t)$.	Factoring $\frac{dP}{dt} = 1200P - 4P^2 = 1200P\left(1 - \frac{P}{300}\right)$ we can see that $\frac{dP}{dt} = 0$ when P=0 and P=300. Therefore, P=300 is the $\lim_{n \to \infty} P(t)$ since the grow increases between P = 0 and P = 300 but stops at P = 300.
9. Given that a line segment has endpoints of (1,2) and (5,10), write a set of parametric equations for the line that passes through these two points.	Determine the slope (m) of the line segment (m=2), write an equation for the line segment using point slope form (y=2(x-1)+2), and then rewrite this equation as parametric equations where x(t)=t and y(t)=2(t-1)+2 or y(t)=2t. Select values for t from knowing that x or t starts at 1 and goes to 5 so $1 \le t \le 5$
10. Given the position function of two particles in parametric form, $x_1(t) = f(t), y_1(t) = g(t)$ and $x_2(t) = h(t), y_2(t) = k(t)$, determine if the particles intersect or collide.	For the paths to intersect $x_1(t_1) = x_2(t_2)$ and $y_1(t_1) = y_2(t_2)$. Solve these equations simultaneously to find the time when the paths intersect. For the particles to collide they must be at a point at the same time. Determine the times when each particle is at the given point. If the times match, the particles collide, otherwise their paths only cross.
11. Given a set of parametric equations where $x = f(t), y = g(t)$, find $\frac{dy}{dx}$ or the slope of the tangent line.	Recall that $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

12. A path of a particle is described with a set of parametric equations $x = f(t), y = g(t)$. Find the equation of the tangent line when t = to .	Determine the point where the particle is $(x(t_o), y(t_o))$. Then find the slope of the graph at the time $t=t_o$ by calculating $\frac{dy}{dx} = \left.\frac{dy/dt}{dx/dt}\right\|_{t=t_0}$. Then write the equation of the line in point-slope form.
13. A path of a particle is described with a set of parametric equations $x = f(t), y = g(t)$. a. Find all values of t where the particle's path is vertical. b. Find all values of t where the particle's path is horizontal.	a. Determine the times when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$. b. Determine the times when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$.
14. Given a set of parametric equations where $x = f(t), y = g(t)$, find $\frac{d^2y}{dx^2}$	First find $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ then calculate $\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$.
15. Given the position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$. Find the velocity vector.	Recall that the velocity vector is $v(t) = \langle x'(t), y'(t) \rangle$ which means that you must differential x(t) and y(t) respect to t and then write a vector.
16. The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$. Find the acceleration vector.	Recall that the acceleration vector is $a(t) = \langle x''(t), y''(t) \rangle$ which means that you must differential x'(t) and y'(t) respect to t and then write a vector.
17. The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$. Find the speed of the particle at a moment at time t = a.	Recall that speed is the magnitude of the velocity vector and is found by calculating $\|v(a)\| = \|\langle x'(a), y'(a) \rangle\| = \sqrt{(x'(a))^2 + (y'(a))^2}$
18. Given the velocity vector $v(t) = \langle x'(t), y'(t) \rangle$ and position vector at t = 0 as $\langle x(0), y(0) \rangle$, find the position vector at time t = a.	Recall that the position vector is $\left\langle x(0) + \int_0^a x'(t)dt, y(0) + \int_0^a y'(t)dt \right\rangle$

19. Given $v(t) = \langle x'(t), y'(t) \rangle$ determine when the particle is stopped.	You must consider both x'(t) and y'(t). You need to determine when both x'(t) and y'(t) equal zero.
20. Given $v(t) = \langle x'(t), y'(t) \rangle$ find the slope of the tangent line to the vector at t_1.	You must calculator $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$ and evaluate this expression at t_1.
21. Given a particle moves along a function $y = 3x^2+1$, the rate of change of x or $\frac{dx}{dt} = 3t$ for $t>0$ and $x(0)=1$. Find the particle's position at time t = 3.	Find the change in the x direction or $x(3) = x_o + \int_0^3 x'(t)dt = 1 + \int_0^3 \left(3x^2+1\right)dt = 31$. Determine the y coordinate using the function $y = f(x) = 3(31^2)+1 = 2884$. Write the coordinate: (31, 2884)
22. Find the slope of the tangent line to the polar curve $r = f(\theta)$.	Recall that $x = r\cos\theta, y = r\sin\theta \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$
23. Given a polar curve $r = f\left(\theta\right)$, find horizontal tangents to curve.	Recall that $x = r\cos\theta, y = r\sin\theta$ and then find where $r\sin\theta = 0$ and where $r\cos\theta \neq 0$
24. Find vertical tangents to a polar curve $r = f\left(\theta\right)$.	Recall that $x = r\cos\theta, y = r\sin\theta$ and then find where $r\cos\theta = 0$ and where $r\sin\theta \neq 0$
25. Find the area inside one of the petals on the flower described by $r = 2\cos(3\theta)$.	Recall that one petal can be traced by $-\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}$ and the area can be found by calculating the integral

26. Find the area outside $r_1(\theta)$ but inside $r_2(\theta)$.	First find the points of intersection $\theta = a \ and \ \theta = b$ and then integrate $\int_a^b \frac{1}{2}\left(r_2(\theta) - r_1(\theta)\right)^2 d\theta$
27. Find the arc length of a function $r_1(\theta)$ from $\theta = a \ and \ \theta = b$.	Recall the $x = r\cos\theta$ and $y = r\sin\theta$ to convert from polar form to parametric form. Then use the integral for arc length with parametric equations. Perform the integral $\int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dt}{d\theta}\right)^2}\, d\theta$
28. Find the sum $\frac{3}{2} + \frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \cdots$.	Notice that the sum is a geometric series where $a = \frac{3}{2}$ and $r = \frac{1}{3}$ so the sum is given by $\frac{a}{1-r} = \frac{\frac{3}{2}}{1 - \frac{1}{3}} = 1\frac{1}{2}$
29. Determine if the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{n}{n+1} + \cdots$ converges or diverges	Think about the nth term in this series: $\frac{n}{n+1}$. By the nth term test, since $\lim_{n\to\infty} \frac{n}{n-1} = 1 \neq 0$ the series diverges.
30. Determine if the series $\sum_{n=1}^{\infty} \frac{3}{n+1}$ converges or diverges	Think about using the integral test. $\int_1^{\infty} \frac{3}{x+1} dx = \lim_{b\to\infty} \int_1^b \frac{3}{x+1} dx$ $= \lim_{b\to\infty} \ln \frac{b+1}{2} = \infty$ Therefore, the series diverges since the integral diverges.

31. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ converges or diverges.	One test you might think of using is the p-series test. Since $p = \frac{3}{2} > 0$ converges.
32. Determine if the series $\sum_{n=1}^{\infty} \frac{1+\sin n}{n^2}$ converges or diverges.	Since the series $\sum_{n=1}^{\infty} \frac{2}{x^2}$ converges and $0 < \frac{1+\sin x}{x^2} \leq \frac{2}{x^2}$ the series converges by the comparison test.
33. Determine if the series $\sum_{n=1}^{\infty} \frac{3^n}{4^n+1}$ converges or diverges.	Using the ratio test, $\lim_{n\to\infty} \frac{\frac{3^{n+1}}{4^{n+1}+1}}{\frac{3^n}{4^n+1}} = \frac{3}{4^{n+1}+1}\left(4^n+1\right) < \frac{3}{4^{n+1}}\left(4^n\right) = \frac{3}{4} < 1$ So the series converges.
34. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ converges or diverges.	Using the alternating series test, since each term $\frac{1}{n}$ decreases as n approaches infinity and converges to 0 , then the alternating series converges.
35. Write a series for $x \cos x$ where n is an integer	Recall that $\cos x = 1 - \frac{x^2}{2\,!} + \frac{x^4}{4\,!} - \frac{x^6}{6\,!} + \ldots$ and then multiply through by x.
36. Write a series for $\ln(1+3x)$ centered at $x = 0$.	Recall that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots$ Substitute 3x for x.

37. If $f(x) = 2 + x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \ldots$ represents a T Taylor Polynomials about x = 0, find $f(-2)$	Notice that f(x) is a geometric series with a = 2 and r = $\frac{1}{2}x$ so the sum of the series can be written as $\frac{a}{1-r}$ or $\left.\frac{2}{1-\frac{1}{2}x}\right\|_{x=-2} = \frac{2}{1-\frac{1}{2}(-2)} = 1$
38. Write the nth degree Taylor Polynomial for f(x) at x = c.	$T_n(x) = f(c) + f'(c)(x-c) + f''(c)\frac{(x-c)^2}{2!} + \cdots + f^{(n)}(c)\frac{(x-c)^n}{n!} + \cdots$
39. Given a Taylor series, find the Lagrange form of the remainder for the 4th term.	This error is no greater than the value of the 5th term at some value of a between x and c. $R_n(x) = \frac{f^{(n+1)}(a)}{(n+1)!}(x-c)^{n+1}$
40 Let S_4 be the sum of the first 4 terms of an alternating series for f(x). Approximate $\|f(x) - S_4\|$.	You should recognize this as the error for the 4th term of an alternating series which is no greater than the absolute value of the 5th term.
41. Given the polynomials $f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$, what is f(x)?	$f(x) = e^x$
42. Given the polynomial $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \ldots$, what is f(x)?	f(x) = sin (x)
43. Given the polynomial $f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + \ldots$, what is f(x)?	f(x) = cos (x)

44. Find the interval of convergence of a series.	Apply the ratio test to find the interval and then test convergence at the endpoints.
45. Find $\lim_{x \to a} \frac{f(x)}{g(x)}$	Check to see if you can use L'Hopital's Rule. Check to see if $f(a) = g(a) = 0$ or $f(a) = g(a) = \infty$. If this is true, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$. You may have to repeat these steps.
46. Find $\int \frac{dx}{x^2 + x - 12}$	Use partial fraction to set up two integrals: $\int \frac{A}{x+4} dx + \int \frac{B}{x-3} dx$. Solve for A and B and then complete the integration. $\int \frac{-\frac{1}{7}}{x+4} dx + \int \frac{\frac{1}{7}}{x-3} dx = \frac{1}{7} \ln \left\| \frac{x-3}{x+4} \right\| + C$