4-2 Conservation of Momentum

According to the **law of conservation of momentum,** the total momentum in a system remains the same if no external forces act on the system. Consider the two types of collisions that can occur.

Vocabulary

Elastic collision: A collision in which objects collide and bounce apart with no energy loss.

In an elastic collision, because momentum is conserved, the *mv* before a collision for each of the two objects must equal the *mv* after the collision for each of the two objects. This is written as

$$m_1 v_{1o} + m_2 v_{2o} = m_1 v_{1f} + m_2 v_{2f}$$

The subscripts 1 and 2 refer to objects 1 and 2, respectively.

Vocabulary **Inelastic collision:** A collision in which objects collide and some mechanical energy is transformed into heat energy.

A common kind of inelastic collision is one in which the colliding objects stick together, or start out stuck together and then separate. However, in an inelastic collision the objects need not remain stuck together but may instead deform in some way.

Because momentum is also conserved in an inelastic collision, the mv before the collision for each of the two objects must equal the mv after the collision for each of the two objects. When objects are stuck together after the collision (assuming mass does not change), this equation becomes

$$m_1 v_{1o} + m_2 v_{2o} = (m_1 + m_2) v_f$$

where v_f is the combined final velocity of the two objects.

Solved Examples

Example 5: Tubby and his twin brother Chubby have a combined mass of 200.0 kg and are zooming along in a 100.0-kg amusement park bumper car at 10.0 m/s. They bump Melinda's car, which is sitting still. Melinda has a mass of 25.0 kg. After the elastic collision, the twins continue ahead with a speed of 4.12 m/s. How fast is Melinda's car bumped across the floor?

Solution: Notice that you must add the mass of the bumper car to the mass of the riders.

Given: $m_1 = 300.0$ kg
$m_2 = 125.0$ kg
$v_{1o} = 10.0$ m/s
$v_{2o} = 0$ m/s
$v_{1f} = 4.12$ m/s

Unknown: $v_{2f} = ?$
Original equation:
$m_1 v_{1o} + m_2 v_{2o} = m_1 v_{1f} + m_2 v_{2f}$

Solve: $$v_{2f} = \frac{m_1 v_{1o} + m_2 v_{2o} - m_1 v_{1f}}{m_2}$$

$$= \frac{(300.0\text{ kg})(10.0\text{ m/s}) + (125.0\text{ kg})(0\text{ m/s}) - (300.0\text{ kg})(4.12\text{ m/s})}{125.0\text{ kg}}$$

$$= \frac{3000\text{ kg}\cdot\text{m/s} + 0\text{ kg}\cdot\text{m/s} - 1236\text{ kg}\cdot\text{m/s}}{125.0\text{ kg}} = \frac{1764\text{ kg}\cdot\text{m/s}}{125.0\text{ kg}}$$

$= \mathbf{14.1\ m/s}$

Example 6: Sometimes the curiosity factor at the scene of a car accident is so great that it actually produces secondary accidents as a result, while people watch to see what is going on. If an 800.-kg sports car slows to 13.0 m/s to check out an accident scene and the 1200.-kg pick-up truck behind him continues traveling at 25.0 m/s, with what velocity will the two move if they lock bumpers after a rear-end collision?

Solution: Since the two vehicles lock bumpers, both objects have the same final velocity.

Given: $m_1 = 800.\text{ kg}$
$m_2 = 1200.\text{ kg}$
$v_{1o} = 13.0\text{ m/s}$
$v_{2o} = 25.0\text{ m/s}$

Unknown: $v_f = ?$
Original equation:
$m_1v_{1o} + m_2v_{2o} = (m_1 + m_2)v_f$

Solve: $v_f = \dfrac{m_1v_{1o} + m_2v_{2o}}{(m_1 + m_2)} = \dfrac{(800.\text{ kg})(13.0\text{ m/s}) + (1200.\text{ kg})(25.0\text{ m/s})}{(800.\text{ kg} + 1200.\text{ kg})}$

$= \dfrac{10\,400\text{ kg}\cdot\text{m/s} + 30\,000\text{ kg}\cdot\text{m/s}}{2000.\text{ kg}} = \mathbf{20.2\ m/s\ forward}$

Example 7: Charlotte, a 65.0-kg skin diver, shoots a 2.0-kg spear with a speed of 15 m/s at a fish who darts quickly away without getting hit. How fast does Charlotte move backwards when the spear is shot?

Solution: To start, Charlotte and the spear are together and both are at rest.

Given: $m_1 = 65.0\text{ kg}$
$m_2 = 2.0\text{ kg}$
$v_o = 0\text{ m/s}$
$v_{2f} = 15.0\text{ m/s}$

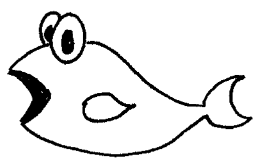
Unknown: $v_{1f} = ?$
Original equation:
$(m_1 + m_2)v_o = m_1v_{1f} + m_2v_{2f}$

Solve: $v_{1f} = \dfrac{(m_1 + m_2)v_o - m_2v_{2f}}{m_1}$

$= \dfrac{(65.0\text{ kg} + 2.0\text{ kg})(0\text{ m/s}) - (2.0\text{ kg})(15\text{ m/s})}{65.0\text{ kg}}$

$= \dfrac{-30.\text{kg}\cdot\text{m/s}}{65.0\text{ kg}} = \mathbf{-0.46\ m/s}$

Remember, the minus sign here is indicating direction. Therefore, Charlotte would travel with a speed of 0.46 m/s in a direction opposite to that of the spear.



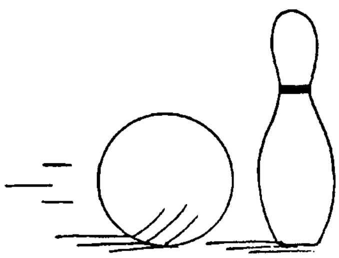
Practice Exercises

Exercise 7: Jamal is at the state fair playing some of the games. At one booth he throws a 0.50-kg ball forward with a velocity of 21.0 m/s in order to hit a 0.20-kg bottle sitting on a shelf, and when he makes contact the bottle goes flying forward at 30.0 m/s. a) What is the velocity of the ball after it hits the bottle? b) If the bottle were more massive, how would this affect the final velocity of the ball?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 8: Jeanne rolls a 7.0-kg bowling ball down the alley for the league championship. One pin is still standing, and Jeanne hits it head-on with a velocity of 9.0 m/s. The 2.0-kg pin acquires a forward velocity of 14.0 m/s. What is the new velocity of the bowling ball?



Answer: ____________________

Exercise 9: Running at 2.0 m/s, Bruce, the 45.0-kg quarterback, collides with Biff, the 90.0-kg tackle, who is traveling at 7.0 m/s in the other direction. Upon collision, Biff continues to travel forward at 1.0 m/s. How fast is Bruce knocked backwards?

Answer: ____________________

Exercise 10: Anthony and Sissy are participating in the "Roll-a-Rama" rollerskating dance championship. While 75.0-kg Anthony rollerskates backwards at 3.0 m/s, 60.0-kg Sissy jumps into his arms with a velocity of 5.0 m/s in the same direction. a) How fast does the pair roll backwards together? b) If Anthony is skating toward Sissy when she jumps, would their combined final velocity be larger or smaller than your answer to part a? Why?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 11: To test the strength of a retainment wall designed to protect a nuclear reactor, a rocket-propelled F-4 Phantom jet aircraft was crashed head-on into a concrete barrier at high speed in Sandia, New Mexico on April 19, 1988. The F-4 phantom had a mass of 19100 kg, while the retainment wall's mass was 469000 kg. The wall sat on a cushion of air that allowed it to move during impact. If the wall and F-4 moved together at 8.41 m/s during the collision, what was the initial speed of the F-4 Phantom?

Answer: ____________________

Exercise 12: Valentina, the Russian Cosmonaut, goes outside her ship for a spacewalk, but when she is floating 15 m from the ship, her tether catches on a sharp piece of metal and is severed. Valentina tosses her 2.0-kg camera away from the spaceship with a speed of 12 m/s. a) How fast will Valentina, whose mass is now 68 kg, travel toward the spaceship? b) Assuming the spaceship remains at rest with respect to Valentina, how long will it take her to reach the ship?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 13: A 620.-kg moose stands in the middle of the railroad tracks, frozen by the lights of an oncoming 10 000.-kg locomotive that is traveling at 10.0 m/s. The engineer sees the moose but is unable to stop the train in time and the moose rides down the track sitting on the cowcatcher. What is the new combined velocity of the locomotive and the moose?

Answer: ____________________

Exercise 14: Lee is rolling along on her 4.0-kg skateboard with a constant speed of 3.0 m/s when she jumps off the back and continues forward with a velocity of 2.0 m/s relative to the ground. This causes the skateboard to go flying forward with a speed of 15.5 m/s relative to the ground. What is Lee's mass?

Answer: ____________________

7. a) 9.0 m/s
9. 10. m/s
11. 215 m/s
13. 9.42 m/s